Physics 238

Alternating Current Circuits and Filters II
Due 1:15 p.m. Friday, May 2, 2025

1 INTRODUCTION

In this experiment you will examine the behavior of high pass and low pass filters from a Fourier perspective. These circuits have a response that depends on the frequency of the incoming signal. In the spirit of Fourier analysis, you will then look at what happens to complex signals that are a superposition of signals at many frequencies.

1.1 Notation Conventions

We will typically use lower case letters, e.g. v, for time-varying quantities, and upper case letters, e.g. V, for constant quantities. Of course, sometimes those "constant" quantities themselves will be slowly varying in time, so the distinction is not absolute. Typically, we will consider an input signal of the form

$$v_{in}(t) = V_{in} \cos \omega t \,,$$

and the corresponding output signal will be of the form

$$v_{out}(t) = V_{out} \cos(\omega t + \phi) \,.$$

The constants V_{in} and V_{out} are known as amplitudes. The ratio $A = V_{out}/V_{in}$ is known as the voltage gain. Even though A is less than 1 in the absence of amplification of some sort, it is still called the voltage "gain".

2 Theory

The main idea is that a periodic signal $f(t)$ can be represented by a Fourier Series of sine and cosine terms:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos\left(2\pi\, n\, f t\right) + b_n \sin\left(2\pi\, n\, f t\right) \right)$$

where T is the period of the signal, $f = 1/T$ is the fundamental frequency, and the coefficients a_n and b_n can be found by

$$a_n = \frac{2}{T} \int_0^T \cos\left(2\pi\, n\, f)\, t\right) f(t) dt$$
$$b_n = \frac{2}{T} \int_0^T \sin\left(2\pi\, n\, f)\, t\right) f(t) dt$$

That is, the signal consists of a "fundamental" at frequency f, as well as higher harmonics at frequencies $2f$, $3f$, $4f$, *etc.* Two specific signals we will consider are the triangle wave and the square wave.

A triangle wave of amplitude A and fundamental frequency f can be presented by

$$\text{triangle}(t) = \frac{8A}{\pi^2}\left[\sin(2\pi f t) - \frac{1}{3^2}\sin(3\times(2\pi f t)) + \frac{1}{5^2}\sin(5\times(2\pi f t))\ldots\right]$$

Note that only odd harmonics are present. For example, if the fundamental frequency is 5000 Hz, with an amplitude of 1 V, then there will also be components at $3 \times 5000\,\text{Hz} = 15\,000\,\text{Hz}$ with an amplitude of $\frac{1}{9}$V, $5 \times 5000\,\text{Hz} = 25\,000\,\text{Hz}$, with an amplitude of $\frac{1}{25}$V, *etc.*

A square wave of amplitude A and fundamental frequency f can be presented by

$$\text{square}(t) = \frac{4A}{\pi}\left[\sin(2\pi f t) + \frac{1}{3}\sin(3\times(2\pi f t)) + \frac{1}{5}\sin(5\times(2\pi f t))\ldots\right] \quad (1)$$

Note that for the square wave, the amplitudes of successive terms fall of as $1/n$, while for the triangle wave, they fall off as $1/n^2$. This means that the higher harmonics are more prominent in a square wave, reflecting the fact that the square wave has sharper transitions.

3 Initial Measurements

For this experiment, you will use a Tektronix TBS 1000C or TDS 2012c digital oscilloscope. These oscilloscopes have a built-in fourier transform mode (FFT) that will enable us to quickly analyze the response of various circuits. The user manual for the oscilloscope is available in the lab. There is also a `Help` button on the front panel.

Throughout this experiment, you may ignore uncertainties.

3.1 Voltage Measurements

Plug the Hi-Ω output of the function-generator into the Ch 1 input of the oscilloscope. Set the function generator to 5000 Hz sine waves, and put the amplitude about half-way to the maximum.

Note that on the right-hand side of the screen there is space for a menu. The menu available is selected by pressing the appropriate button on the front panel. For example, you can select the `CH 1` or `CH 2` menu, or `Measure` or `Cursor`. The Fast Fourier Transform (or FFT) option is either a separate button or available under the `Math` menu.

3.2 Saving Data and Images

There are two main options for saving data and images from the oscilloscope. First, you can plug a USB drive into the front panel of the oscilloscope and save images and data directly there. See the instruction manual under "Data Logging" for more details.

Second, the oscilloscope can be connected to the computer via a USB cable. Launch the `OpenChoiceDesktop` program. This program can

1. Capture screen images and save as PNG files, suitable for import into Word or LaTeX.
2. Save raw Ch 1 and Ch 2 data (you need to select Ch 2 on the channels menu) as a "Comma Separated Values" (csv) file, suitable for import (or even pasting) into `Excel` (or *Mathematica*, if you prefer, but it might be easier to clean up the columns in `Excel`).

3.3 FFT

The oscilloscope can measure both the amplitude and frequency of the different Fourier components of a signal.

3.3.1 Sine Wave

Make sure the function generator is set to 5000 Hz sine waves. Use the oscilloscope to record the amplitude of the signal. Select the FFT option. Displayed on the screen will be a power spectrum. Because you have a nice clean sine wave, there is only one significant peak. The default settings leave this peak almost undetectable against the left-hand edge of the screen. Adjust the horizontal Time/cm knob until the peak is more easily visible. A setting of 12.5 kHz (on the TDS 2012C) or 125 kS/s (on the TBS100C) works well. For the amplitude measurements relevant for this type of experiment, the `Flattop` or `Rectangular` window typically works well. (See also Fig. 1 below for an example for an triangle wave, and Fig. 2 for a square wave.)

Use the `Cursor` to move a cursor along the screen and measure the frequency corresponding to the peak. It should be 5000 Hz. Also record the amplitude of the peak. Note that the amplitudes are given in decibels, which is a logarithmic scale. That is,

$$dB = 20 \log_{10} \left(\frac{V}{V_{\text{ref}}} \right)$$

where V_{ref} is taken to be 1.0 V. (Different reference voltages are used in different contexts.) Is the amplitude of the FFT peak consistent with the amplitude you measured for the sine wave?

3.3.2 Triangle Wave

Now set the function generator to produce 5000 Hz triangle waves. Record the amplitude of the wave from the oscilloscope. Switch to the FFT display. You should see something like Fig. 1.

As you may recall, the triangle wave includes all of the odd harmonics, but the amplitudes of those harmonics decrease as $1/n^2$. You should see peaks at 5, 15, 25, 35, 45, ... kHz, but those peaks should get progressively smaller according to $1/n^2$. Use the `Cursor` to measure individual peaks. Specifically, note the frequencies and heights of the first five main peaks in the spectrum. Do they agree with the expected values?

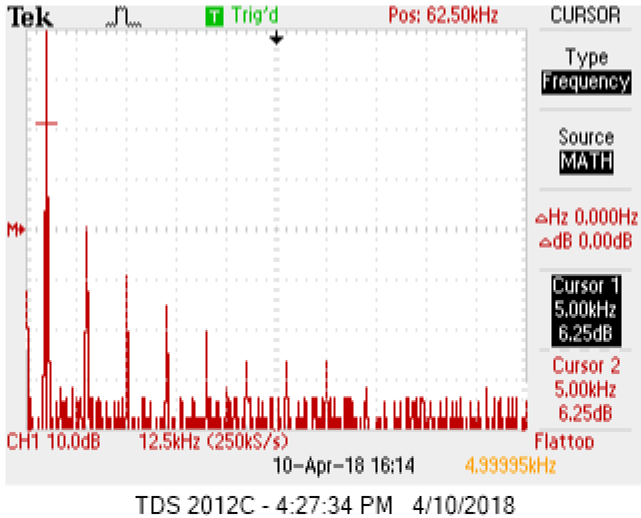

Figure 1: Oscilloscope display for FFT of an input 5000 Hz triangle wave on the TDS2012C oscilloscope. (See Fig. 2 for a similar image for the TBS-1000C oscilloscope.) Both cursor 1 and cursor 2 are set on the fundamental, which has a frequency of 5.0 kHz and an amplitude of 6.25 dB. Note the successive peaks at 15.0 kHz, 25.0 kHz, *etc.* These amplitude measurements were made with the `Flattop` window.

3.3.3 Square Wave

Lastly, set the function generator to produce 5000 Hz square waves. For the FFT display, you should see something like Fig. 2.

As you may recall, the square wave includes all of the odd harmonics, and the amplitudes of those harmonics decrease as $1/n$. Thus you should again see peaks at 5, 15, 25, 35, 45, ... kHz, but those peaks should not decay as quickly as they did for the triangle wave.

Note that the square wave display is very noisy. (The function generator can not produce ideal square waves, and the oscilloscope couldn't handle a true discontinuity exactly anyway.) Nevertheless, you ought to be able to make out the main peaks. There will be additional (smaller) peaks at other frequencies, but you can ignore them.

Record the frequencies and heights of the first five main peaks in the spectrum. Do they agree with the expected values?

Make a brief qualitative comparison between the triangle and square waves. (You can toggle the function generator back and forth to make a quick visual comparison.) How the the spectra compare? Which one drops off more rapidly? Is that consistent with the Fourier series expectations?

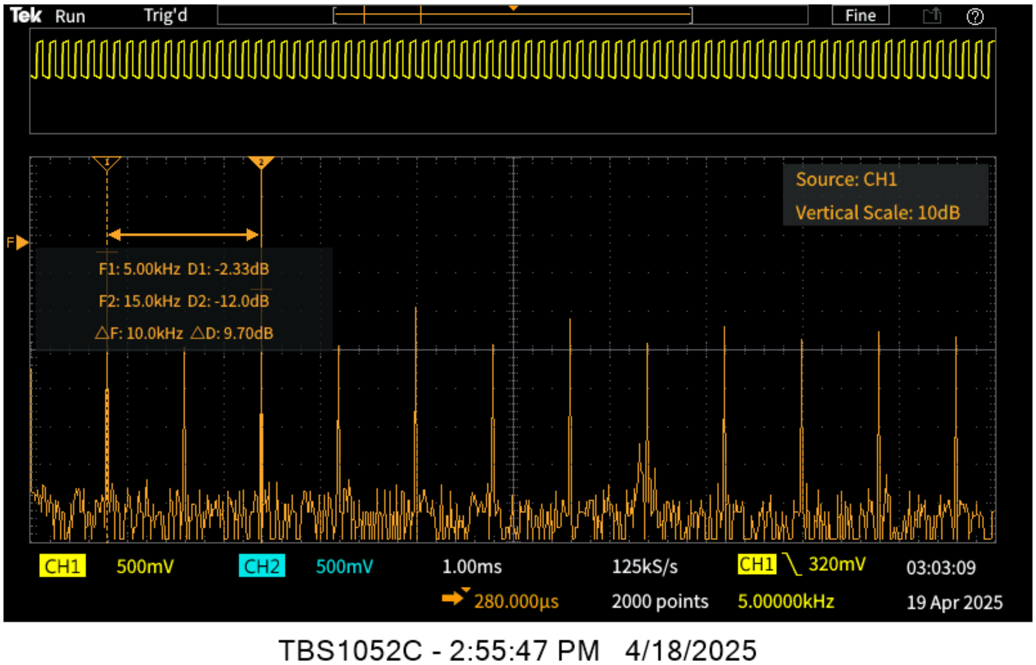

Figure 2: Oscilloscope display for FFT of an input 5000 Hz square wave. Cursor 1 is set on the fundamental, which has a frequency of 5.0 kHz and an amplitude of −2.33 dB. Cursor 2 is set at 15.0 kHz and has an amplitude of −12.0 dB. Note the successive peaks at 15.0 kHz, 25.0 kHz, *etc.* Since the wave produced by the function generator is not an ideal square wave, there are also smaller peaks at even harmonics, 10.0 kHz, 20.0 kHz, *etc.* These amplitude measurements were made with the `Rectangular` window.

4 RC Low Pass Filter

Construct the circuit shown in Fig. 3. Use $R = 3.3\,\mathrm{k\Omega}$ and $C = 22\,\mathrm{nF}$.[1] These are nominal values only. *Measure, record, and* use *the actual resistance and capacitance values.* (These are the same components you used for the AC filters experiment.) Set the function generator to sine waves.

4.1 Theory

The amplitude of the output voltage is given by $V_{out} = V_{in}\left(\frac{Z_C}{Z}\right)$, where $Z_C = 1/\omega RC$ is the impedance of the capacitor, and Z is the total impedance, given by

$$Z = \sqrt{R^2 + \frac{1}{(\omega C)^2}}\,.$$

The voltage gain $A(f)$ of the circuit is defined to be $V_{\text{out}}/V_{\text{in}}$, and is given for this circuit

[1]Remember that the capacitor will likely be labeled "223" for $22 \times 10^3\,\mathrm{pF} = 22\,\mathrm{nF} = 0.022\,\mathrm{\mu F}$.

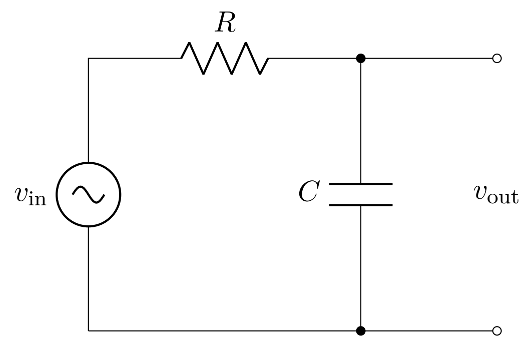

Figure 3: RC High Pass Filter.

by

$$A(f) = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{Z_C}{Z} = \frac{1}{\sqrt{1+(2\pi fRC)^2}}\ , \tag{2}$$

where the last expression follows from some algebra. At small f the gain is near 1, but as the frequency increases, the gain falls and approaches 0. The "breakpoint" frequency is defined to be $f_b = \frac{1}{2\pi RC}$. At this frequency, A has decreased to a value $1/\sqrt{2}$, and the power has decreased to 1/2 of its peak value.

The phase difference between v_{in} and v_{out} is given by

$$\tan\phi = -2\pi fRC$$

Set up the oscilloscope to display both Ch. 1 and Ch. 2. Monitor the input voltage on Channel 1, and the output voltage (The voltage across C) on Channel 2. Use the 'DC' settings for both channels. (All that the AC setting does is (effectively) subtract out the average value of the incoming signal.)

Calculate the expected linear breakpoint frequency $f_b = 1/(2\pi RC)$ for your circuit.

4.2 Sine Wave Response

4.2.1 Voltage Gain and Phase Difference

(For this section, you don't have to record any data. These are simply some quick checks to verify that the circuit is set up correctly and you are seeing what you'd expect for a low pass filter.)

Set your function generator to a frequency far below f_b (e.g. $f_b/10$). Set the vertical scales for Ch. 1 and Ch. 2 to the same setting. For the low-pass filter, you should have a gain very close to 1, and a phase shift very close to 0. Does that match what you measure on the oscilloscope? Now check the response of your circuit to a frequency close to f_b. Do you get the expected values for A and ϕ? Lastly, check the response of your circuit for a very high frequency (e.g. $10f_b$). Do you get the expected values for A and ϕ?

4.2.2 FFT

Now set your oscilloscope to FFT mode. Compare the FFT of the *input* (*i.e.* Ch. 1) and *output* (*i.e.* Ch. 2) signals for low, medium, and high frequencies. You don't have to make any quantitative measurements, but do record what you observe about the main peak in the spectrum. Note that you only have amplitude information; the FFT display doesn't tell you anything about the phase.

4.3 Triangle Wave Response

In this section, you will explore how low, medium, and high frequency triangle waves are affected by the low pass filter. Frequencies of about $f_b/10$, f_b, and $10f_b$ work well.

4.3.1 Experiment

Set the oscilloscope back to the normal dual-trace voltage *vs.* time mode. Set the input signal to a low frequency triangle wave with an amplitude of about 1 V. It is probably easiest to use the same vertical scales on Ch. 1 and Ch. 2, and to have both traces centered vertically on the screen. Record an image showing the input and output waves. Next, switch to FFT mode and measure the frequency and amplitude of the first five main peaks of the FFT spectrum for both the *input* and *output* signals.

Repeat these measurements for medium and high frequency triangle waves.

4.3.2 Analysis

FFT Amplitudes Compare your FFT amplitudes with those predicted for a low pass filter. Specifically, for the low frequency triangle wave, plot the amplitude of the *output* FFT peaks *vs.* frequency. On the same graph, plot your *predictions* for those amplitudes based on the *input* amplitudes passed through Eq. 2. Produce similar graphs for the medium and high frequency triangle waves.

Discussion of Triangle Wave Look back at the output signals you got for low, medium, and high frequency triangle waves. Discuss how the those shapes result from the behavior of the individual frequency components of the triangle wave passing through a low pass filter. Refer to your FFT results as necessary.

4.4 Square Wave Response

Switch to an input square wave and repeat the measurements, analysis, and discussion of §4.3.

4.5 Response to Noise

Finally, consider what happens if you send in a noisy signal instead of a sine, square, or triangle wave. You can use either one of the GWInstek function generators or the TeachSpin Signal Processor / Lock-in Amplifier. For the TeachSpin Signal Processor, at the lower left, there is a "noise generator." Do not connect anything to the input `Signal` terminal. Use the signal coming from the `Signal + Noise` output terminal as the *input* to your low-pass filter. Turn the `Noise Amplitude` all the way up to `1`.

Look at the input signal (Ch. 1) on your oscilloscope. Expand the vertical scale so the signal fills most of the screen. Now set the output (Ch. 2) vertical scale to match. Changing the horizontal scale shouldn't make much visible difference, but a value of 1.00 ms per division is sensible. You should have a very noisy signal. Now look at it in the FFT display. (A horizontal scale of about 12.5 kHz or 125 kS/s works well, though feel free to explore other ranges.) Record images of the FFT of the input and output signals. How does the output compare to what you'd expect for your low-pass filter? You need not make any quantitative measurements here, but do discuss whether the general behavior you see is consistent with what you'd expect for a low pass filter.