# Least Squares Fitting -- Residuals: Physics 238

In[55]:=
```
Clear["Global`*"]; DateString[]
```
Out[55]=
```
Mon 17 Feb 2025 11:23:07
```

## Data

In[56]:=
```
SetDirectory[NotebookDirectory[]]
```
Out[56]=
```
/Users/doughera/238/2025/lectures-dev/Ch08
```

In[57]:=
```
FilePrint["TO-static-20250212.csv", 5]
```
```
"Mass (g)", "Raw Angle", "Voltage"
0, 2.98, -0.0456
50, 2.79, -0.4083
100, 2.59, -0.7733
150, 2.38, -1.172
```

File is a CSV file, with 3 columns. We just want columns 1 and 2.

In[58]:=
```
rawdata = Import["TO-static-20250212.csv", "CSV"];
```

In[59]:=
```
data3 = Select[rawdata, NumberQ[#[[1]]] && NumberQ[#[[2]]] &];
```

For this data, we want columns 1 and 2.

In[60]:=
```
data = Table[{data3[[i, 1]] , data3[[i, 2]]}, {i, 1, Length[data3]}]
```
Out[60]=
```
{{0, 2.98}, {50, 2.79}, {100, 2.59}, {150, 2.38},
 {200, 2.17}, {250, 1.96}, {300, 1.66}, {350, 1.41}, {400, 1.21},
 {0, 2.98}, {-50, 3.2}, {-100, 3.4}, {-150, 3.6}, {-200, 3.81},
 {-250, 4.06}, {-300, 4.32}, {-350, 4.56}, {-400, 4.74}, {0, 3.}}
```

In[61]:=
```
dataPlot = ListPlot[data,
   LabelStyle → Larger,
   AxesLabel → {"Mass (g)", "Angle (radians)"}, ImageSize → Scaled[0.7]];
```

In[62]:=
```
fit = LinearModelFit[data, x, x]
```
Out[62]=
```
FittedModel[ 2.99 − 0.00436 x ]
```

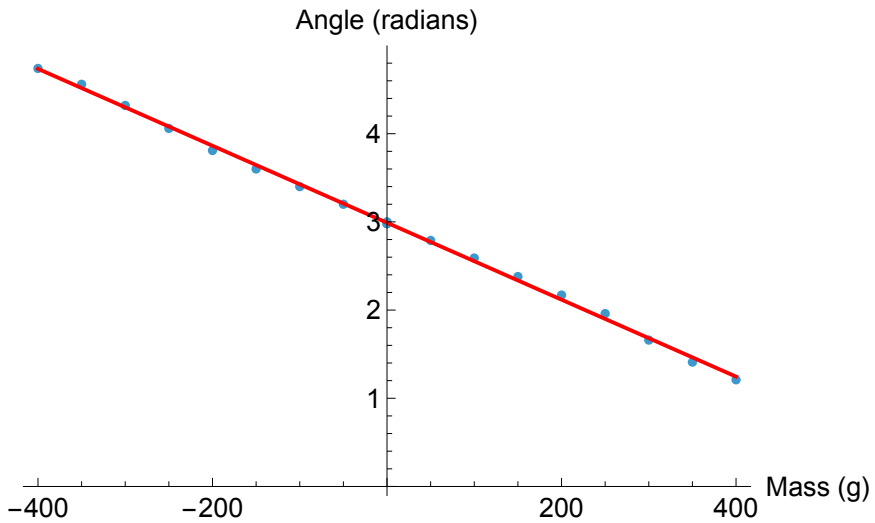
In[63]:=
```
fit["ParameterConfidenceIntervalTable"]
```

Out[63]=

| | Estimate | Standard Error | Confidence Interval |
|---|---|---|---|
| 1 | 2.99053 | 0.00845294 | {2.97269, 3.00836 } |
| x | −0.00436275 | 0.0000364825 | {−0.00443972 , −0.00428577 } |

In[64]:=
```
Show[{dataPlot, Plot[fit[x] , {x, -400, 400}, PlotStyle → Red]}]
```

Out[64]=



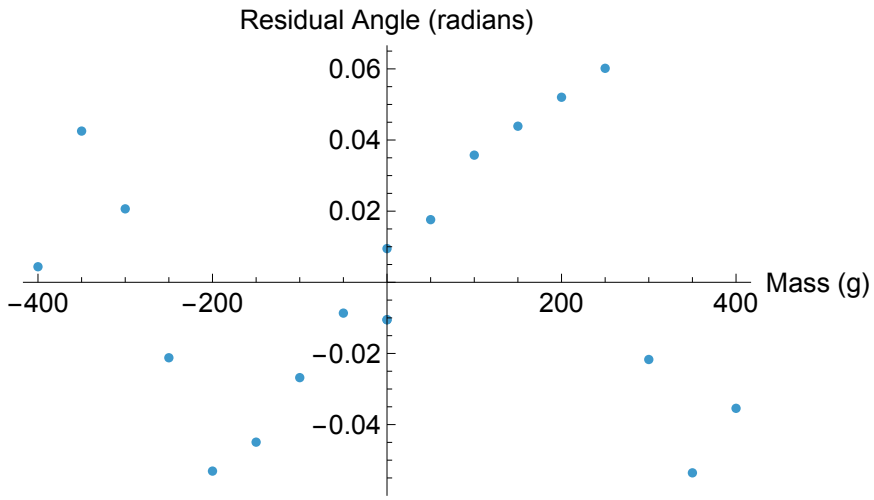

In[65]:=
```
residuals =
 Table[{data[[i, 1]], data[[i, 2]] - fit[data[[i, 1]]]}, {i, 1, Length[data]}]
```

Out[65]=

```
{{0, -0.0105263}, {50, 0.0176109}, {100, 0.0357482}, {150, 0.0438854},
 {200, 0.0520227}, {250, 0.06016}, {300, -0.0217028}, {350, -0.0535655},
 {400, -0.0354283}, {0, -0.0105263}, {-50, -0.00866357}, {-100, -0.0268008},
 {-150, -0.0449381}, {-200, -0.0530753}, {-250, -0.0212126},
 {-300, 0.0206502}, {-350, 0.0425129}, {-400, 0.00437564}, {0, 0.00947368}}
```

In[66]:=
```
ListPlot[residuals, PlotRange → All, LabelStyle → Larger,
 AxesLabel → {"Mass (g)", "Residual Angle (radians)"}, ImageSize → Scaled[0.7]]
```

Out[66]=




In[67]:=
```
RMSE = Sqrt[fit["EstimatedVariance"]]
```

Out[67]=
```
0.0368455
```

The RMS error is plausible, but there is a clear systematic trend in the residuals suggesting the data may behave differently for larger masses.

# Limiting data to smaller torques

Bundle up all those commands into a single cell to consider different limits on data. Select only a more narrow range of masses.

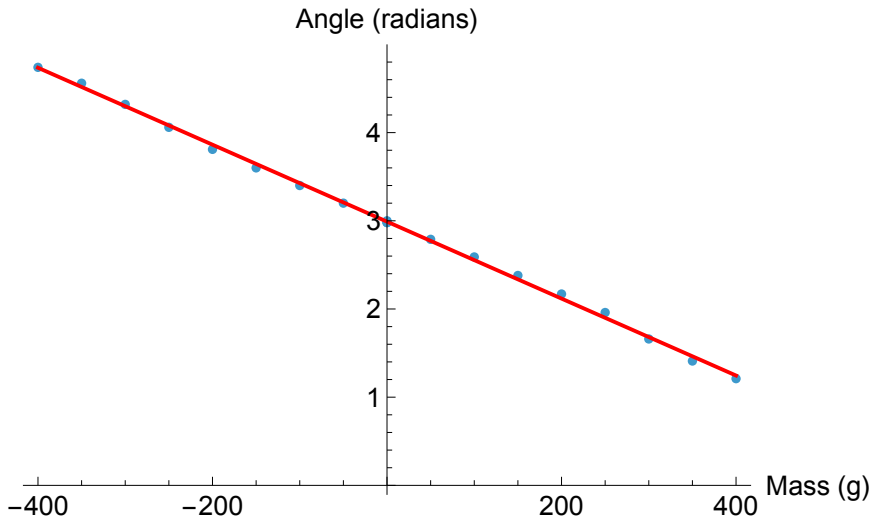
In[68]:=
```
fitdata = Select[data, -400 ≤ #[[1]] ≤ 400 &];
dataPlot = ListPlot[data,
   LabelStyle → Larger,
   AxesLabel → {"Mass (g)", "Angle (radians)"}, ImageSize → Scaled[0.7]];
fit = LinearModelFit[fitdata, x, x]
fit["ParameterConfidenceIntervalTable"]
Show[{dataPlot, Plot[fit[x] , {x, -400, 400}, PlotStyle → Red]}]
residuals =
  Table[{data[[i, 1]], data[[i, 2]] - fit[data[[i, 1]]]}, {i, 1, Length[data]}];
ListPlot[residuals, PlotRange → All, LabelStyle → Larger,
 AxesLabel → {"Mass (g)", "Residual Angle (radians)"}, ImageSize → Scaled[0.7]]
Sqrt[fit["EstimatedVariance"]]
```

Out[70]=
```
FittedModel[ 2.99 − 0.00436 x ]
```

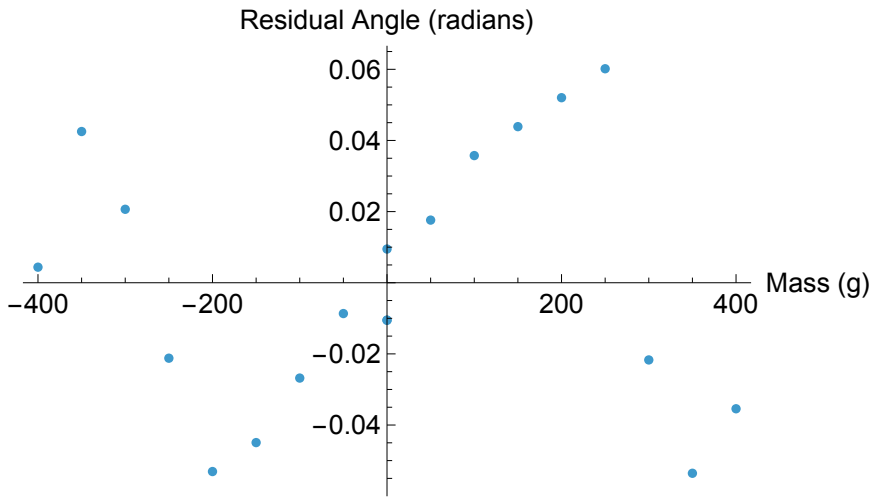
Out[71]=

| | Estimate | Standard Error | Confidence Interval |
|---|---|---|---|
| 1 | 2.99053 | 0.00845294 | {2.97269, 3.00836 } |
| x | −0.00436275 | 0.0000364825 | {−0.00443972 , −0.00428577 } |

Out[72]=




Out[74]=




Out[75]=

```
0.0368455
```