Physics 238

The Simple Pendulum

Estimating, Propagating, and Understanding Uncertainty

Report: **Due Friday, February 7, 2025**

Safety

The "knife edge" used for the top end of the pendulum is a sharp razor blade. *Be careful.* (The knife edge ensures a clean pivot with a very well-defined position.)

1 Introduction

The local acceleration of gravity, g, is an important parameter for many aspects of geophysics and planetary science. Slight variations in g across the surface of a planet provide clues about underlying geological features, the existence of mineral deposits, or the presence of subsurface water. For such measurements to be useful, however, it is important to have a good understanding of the uncertainties involved.

In this experiment, you will use a simple pendulum to estimate the value of g, the local acceleration due to gravity. Along the way, you will also study distributions, the mean, standard deviation, and standard deviation of the mean, random and non-random error, and the estimation, propagation, and interpretation of uncertainties.

The main experiment will follow the example in section 3.9 (pg. 68) in Taylor's text. It is worth reading that example carefully. The normal distribution and the statistical interpretation of data are covered in chapters 4 and 5.

2 Theory

The period of a simple pendulum for small oscillations is given by

$$T = 2\pi\sqrt{\frac{L}{g}}\ , \tag{1}$$

where L is the pendulum length. By measuring L and T, you can determine g. Careful measurements of T for different amplitudes of oscillation will also help you determine whether the small oscillation approximation was justified. You will also be able to determine if the "simple pendulum" approximation was justified.

Note: Remember these approximations as you acquire and analyze your data. Some of your results may only make sense if you review whether these approximations were justified. You may even be able to alter your experiment so that they are better satisfied, or refine the analysis to compensate.

3 Experiment

3.1 Keeping Good Records

It is important to keep good records for any experiment. There is no specific lab notebook requirement for this course, but it is important that you be able to find and use your data even after you leave the lab. For this experiment, you will need to use your data and observations for the first homework assignment in addition to the lab report.

Some students may find it handy to use a written notebook, while others may prefer to use on-line documents. Either way is fine for this course. Do not just write data on random scraps of paper that can easily be lost. Different professional labs and groups will have different specific standard practices, but will share the common theme of storing your data safely so that it can be used again later, both by yourself and by your colleagues.

3.2 Initial Data Acquisition

Set up a simple pendulum. Measure L. Make an estimate of your uncertainty in L as well. *Be careful*: It is critical that you do this step well! Make sketches or detailed descriptions of the pendulum bob and of the clamp to indicate precisely what you measured. Record the height of the bob and the length of the string. Take good notes here. You will need this information for a later homework assignment.

Launch `LoggerPro` and use the `File` → `Open` menu item to load in the `Probes and Sensors` → `Photogates` → `Pendulum Timer` experiment.[1] After you have experimented around for a bit to see how it works, use the `Experiment -> Data Collection` menu item to set the `Duration` to 40 s and let the program record the period for 20 successive swings. Don't touch it during this time. Save your data.

3.2.1 A First Look

Adjust the scales on your graph so that the data take up most of the graph. (The autoscale option (the big "A" on the toolbar) usually works well for this.) Do your data look reliable? Are there any significant issues or systematic trends? Try to resolve any issues before proceeding. There is little point in continuing with bad data. Review the note at the end of the theory section above. *Fix any problems now.* It is far easier to do another 40 s trial now than to spend a lot of time trying to clean up poor data later.

3.2.2 Mean, Standard Deviation, and Uncertainty of the Mean

Consult Ch. 4 in Taylor and calculate the mean, standard deviation, and uncertainty (also known as standard deviation of the mean) for your data. You can easily do this in `LoggerPro` with the `Analyze` → `Statistics` menu option (It will compute the mean and

[1] If `LoggerPro` prompts you to connect to the photogate, hit the `Connect` button.

standard deviation; you still need to manually compute the uncertainty of the mean by taking $\sigma/\sqrt{N}$.) Note that the printed box might not contain enough significant digits. Right-click on the statistics box, select `Statistics Options`, and increase the number of significant figures or decimal places as needed. (Remember that you typically round off the uncertainty to one digit, and then round off the mean to match.) While in that menu, you should also click on the `Appearance` button and increase the font size to at least 14 so the box is still readable when you print it out.

What percent of your data points lie less than ± 1 standard deviation from your mean value? Is that a reasonable result? (No sophisticated analysis is required here, just qualitative estimates.)

Make a screenshot of your final graph, including the statistics. Include it in your final report.

3.3 Extensive Data Acquisition

Now set up `LoggerPro` to record the period for about 250 oscillations (about 500 seconds). During this time, be careful not to move around the room. The timing is very sensitive to minor air currents, and moving around will invalidate at least some of your data, and also the data of any other students nearby. Save your work. Do not print out the data table. Also export your data to a text file: Choose `File` → `Export` and save it as a "CSV" (comma-separated-values) file. This format can then be imported by many other programs.

Important: Leave your experiment set up until you are certain *that you are done and have completed all of the analysis!* You may leave it set up until you turn in your final report.

3.3.1 Troubleshooting Tips

Symptom: T steadily decreases. Likely cause: The initial amplitude was too large. As the pendulum motion is damped, the amplitude (and hence period) decreases. Try a smaller amplitude.

Symptom: T oscillates: high – low – high – low. Likely cause: The pendulum may be oscillating in an ellipse rather than a single plane, may be wobbling, or the photogate might not be centered on the equilibrium position. Try centering the photogate and give the pendulum a small gentle oscillation perpendicular to the knife-edge holding the string at the top. Sometimes just waiting for the motion to settle a bit more will help.

Symptom: Two successive points are wildly out of line (one high / one low). Likely cause: The pendulum was perturbed slightly, perhaps by someone walking near it. Ignore. You can delete those two points later.

Symptom: T suddenly doubles. Likely cause: The amplitude is so small that the pendulum isn't fully clearing the light beam on the photogate. If possible, try raising the support bar (or lowering the photogate) a little so that just the tip of the pendulum breaks the beam. If necessary, increase the amplitude slightly. If this only happens at the very end of your data acquisition, you can just delete those bad points and use the remaining data points.

3.3.2 A First Look

Look at the plot of period vs. time. Are there any bad points? (If so, note down why you think they are bad and then delete them.) (Select the data point in the `Period` column and then use the `Edit` → `StrikeOut` menu item.) Do you observe any systematic trends? Is there a way to avoid (or at least minimize) those systematic trends? Make sure your data are reasonable before proceeding to the next step. If you need to get fresh data, do so now.

3.3.3 Mean, Standard Deviation, and Uncertainty of the Mean

Calculate the mean, standard deviation, and uncertainty of the mean for your data. (Use the `Analyze->Statistics` menu item.) Again, make sure the statistics box shows all the relevant digits, and is in a large enough font to read.

Make a screenshot of your final graph, including the statistics.

3.4 Save Your Work

Save your work. Also export your data to a text file: Choose `File` → `Export` and save it as a "CSV" (comma-separated-values) file.

You will need this data for the first homework assignment, so make sure you save it in a place where you can access it later.

4 Final Analysis

At this point, you have measurements for T and L, including uncertainties. Use those values to estimate g. Then, estimate the uncertainty in g. Does your result agree with the expected value of $9.8\,\mathrm{m/s^2}$? If not, try to figure out what went wrong. If necessary, go back to your apparatus and check your measurements for L and T. Reconsider the approximations that went into Eq. 1. *Be persistent.* This experiment should work.

Comment on the relative importance of the different uncertainties in this experiment.