

The Torsional Oscillator—Part 1
Report: Due Friday February 21, 2025

Safety

This experiment involves permanent magnets with strong, but localized, magnetic fields. Just outside the wooden frame of the apparatus, the field is approximately 5 mT (*i.e.* 50 Gauss). This may be large enough to interfere with some sensitive medical devices. If you have any concerns, please consult your lab instructor.

1 Introduction

In this lab, you will perform experiments with a torsional oscillator. You will measure the restoring torsional spring constant κ by both static and dynamic means, and compare the results. Along the way, you will see how to use theoretical fits of experimental data to estimate parameters (and their uncertainties) and how to compare different experimental measurements.

The torsional oscillator comes with an extensive manual. A copy of that manual is in the lab next to the apparatus. The relevant sections are also included with this write-up (and are available on the course website). You should refer to that instruction manual for detailed operating instructions and specifications; this write-up will focus on the higher-level physics questions you should address.

Throughout the experiment, be sure to quote the relevant uncertainties wherever appropriate, and to refer to those uncertainties whenever you make comparisons.

1.1 The Apparatus

Chapter 0 of the instruction manual includes a “walk-through” of the apparatus. It is worthwhile to read carefully through sections 0.0, 0.1, and 0.2. This will familiarize you with the instrument. There is no data to record here, unless you observe something unusual that you think might affect your later results.

2 Static Torque Measurements

In this portion of the experiment, you will apply a static torque τ and measure the resulting angular deflection of the apparatus. By varying the applied torque and measuring the angular displacement, you can then determine the torsional constant κ for the wire. You will also discover how to apply a magnetic torque to the rotor.

2.1 Theory

If there is no applied torque, then the apparatus will be at its relaxed position, θ_R . For a linear system, an applied torque τ will result in an angular displacement given by

$$\tau = -\kappa(\theta_{raw} - \theta_R) \quad (1)$$

where θ_{raw} is the raw angle you read off the apparatus, and κ is the torsional constant with units Nm/rad, and plays a role analogous to a spring constant.

2.1.1 Data Acquisition

The angular displacement can be read either visually from the scale, or electronically from the front panel outputs. In future experiments, we will use the electronic outputs, so this is a good opportunity to calibrate the apparatus and determine how the voltage translates to an angle. Consult section 1.2 of the manual and use **LoggerPro** as a voltmeter to record the output voltage $V_{pos}(\theta)$. *You will not use the voltage in this lab report, but you will need it for the next homework assignment.*

The applied torque τ can be varied by the hanging weights, as described in section 1.0 of the manual. A torque can *also* be applied by supplying current to the Helmholtz coils, as described in sections 2.0 and 2.1 of the manual. In §3 below, this will provide a useful way to set the initial angle.

1. Use a micrometer caliper to measure the diameter of the torsion wire. (The apparatus comes with several wires; the expected diameters are in section 6.1 of the manual, which is also included in this write-up. You may use your measurement to determine which of the four wires you have, and then take the specified value as correct.)
2. Adjust the magnets to apply magnetic damping to the system, so that it settles to equilibrium fairly quickly.
3. Start with no applied torque. Measure the relaxed angle θ_R . Also make a note of your uncertainty in this value – not a statistical analysis, but just an observation of how well you think you can determine this value from the scale.

(Note that the plastic piece in front of the scale has markings on both the front and the back. This is to avoid parallax. Move your head so that the front and back markings are in line with each other, and then read the scale behind the markings. This is a technique to ensure that you are precisely in front of the scale, and not reading it an angle off to the side.)

4. In **LoggerPro**, use the **Experiment**→**Data Acquisition** menu to set the experiment duration to 10s and the rate to 500 samples/second.
5. Adjust the “Zero Adjust” knob so that the output voltage is as close to 0.000 V as you can reasonably get.
6. Use **LoggerPro** to record the voltage for about 5s and then use **Analyze**→**Statistics** to record the actual average value in your notes. This value may drift a bit; that’s ok. You will account for it later. In your notes, you may include an uncertainty to give a rough estimate of how much it appears to drift. *Once you have set the “Zero Adjust” knob, do not adjust it further during this part of the experiment.*
7. Now wind the string as shown in Fig. 1.0b to apply a static torque. For this first trial, just use the 50 g mass hangers on each side. Record the following in a neat table:

The hanging mass on each side, the raw visual angle reading θ_{raw} , and the output voltage (again averaged over 5 seconds) corresponding to that angle. Save room to add 2 more columns to that table, for angular displacements and torque calculations. You don't need to record any uncertainties here.

8. Now vary the torque by symmetrically adding masses to both weight hangers. Keep the total angular deflection under 2 radians. For each torque, record the hanging mass on each side, the raw visual angle, and the output voltage. Again, you don't need to record any uncertainties here.
9. Reverse the direction of the torques and repeat the data collection.
10. Perform a final reading with no torque. This is a good way to see how much hysteresis there is in the apparatus. The effect is likely to be small, but it may be measurable. Record your observations.

2.1.2 Analysis

For each of your trials, compute an angular *displacement* $\theta \equiv \theta_{raw} - \theta_R$, and add that to your table.

Torque

1. Make a graph of θ_{raw} vs. torque. (You should typically place the variable with the greater uncertainties on the vertical axis. In this experiment, that is the angle measurement.) If this is a simple harmonic oscillator, you should expect a relationship of the form given by Eq. 1.
2. Fit Eqn. 1 to your data. Qualitatively, does your data follow such a linear relationship? If not, does it follow a linear relationship over a smaller data range? If so, then explain your findings and confine your fit to that smaller range.
3. What value do you get for θ_R ? Does it agree with the value you measured directly?
4. What value do you get for κ ? Include the uncertainty.

3 Dynamic Measurements

In this section, you will observe the period of oscillation of the device, and use those measurements to find a second estimate for κ .

3.1 Theory

If the restoring torque is linear as in Eq. 1, then the period T of oscillation is related to the moment of inertia I and the torsional constant κ by

$$T = 2\pi\sqrt{\frac{I}{\kappa}}. \quad (2)$$

By varying I , measuring T , and fitting the results to Eqn. 2, you can make an independent determination of κ .

3.2 Experiment

1. Remove the string providing the external torque, and back out the magnetic damping all the way.
2. Connect the DC power supply to the terminals marked “Coil Drive.” *Keep the total current under 2 A!* Slowly turn up the current until $\theta_{raw} - \theta_R$ is about 1 radian, and let it equilibrate.
3. Turn off the DC power supply to start the rotor oscillating with a moderate-sized oscillation. Use **LoggerPro** to record $V(t)$ for at least 5 complete oscillations. Make sure that the sampling rate is sufficiently high to get a good smooth curve. A sampling rate of 500 samples/second works well. Fit a sine function and determine the period of oscillation. (You don’t need to worry about the uncertainties here.)
4. Vary the moment of inertia by adding brass quadrants (two at a time, to maintain balance) to the rotor, as described in the instruction manual, section 1.3. Record the new period.
5. Record the dimensions and mass of the brass quadrants (either measured directly, or from the data in section 6.1 of the instruction manual.)

3.3 Analysis

1. As discussed in the lab manual, the moment of inertia I can be expressed as

$$I = I_0 + n\Delta I,$$

where I_0 is the moment of inertia of the basic apparatus without any brass quadrants, n is the number of brass quadrants you added, and ΔI is the moment of inertia of a single brass quadrant. Rewrite Eqn. 2 in terms of I_0 , n , and ΔI .

2. Calculate ΔI . (See section 6.1 of the instruction manual below.)
3. Plot T vs. n . Fit your data to your revised version of Eqn. 2, and determine κ (and its uncertainty). (Your fit will also give you a value for I_0 , but since you don’t have anything to compare it to, it’s not of further use.)

4 Comparison

4.1 Materials calculation

The restoring torque for a wire of length L and radius r that has been twisted through an angle $(\theta_{raw} - \theta_R)$ is given by

$$\tau = -\frac{\pi Gr^4}{2L}(\theta_{raw} - \theta_R) \quad (3)$$

where G is the shear modulus, which is approximately 80 GPa for the music wire used in this experiment.

Compare Eqs. 1 and 3 to find a theoretical equation for κ . Use the data from section 6.1 of the TeachSpin Instruction Manual (included below) to calculate the expected value of

κ . (The uncertainty in the radius is relevant here, so do include that, but you may ignore all other uncertainties in the material parameters.)

4.2 Final comparison

You now have two different measurements of κ , along with one theoretical value. How do they compare? Do they agree to within the uncertainties? If not, can you determine why not? Do not be afraid to go back to the apparatus and obtain more data!

6.1. Masses and sizes of relevant parts

This section gathers into one place a number of relevant dimensions and masses of various parts of the Torsional Oscillator. Masses are given in grams, and dimensions are quoted (for cultural reasons) in inches, where 1 inch = 1" \equiv 0.0254 m exactly.¹

- The copper rotor disc has maximal outer diameter of 4.95", an inner diameter of 1.02", and total mass of 962 ± 2 g. The diameter of the circle of holes receiving the brass quadrants' dowel pins is $2.720" \pm 0.005"$.
- The rotating part of the angular-position sensor has outer diameter of 4.74", an inner diameter of 1.02", and total mass of 37 ± 1 g. It's made of standard printed-circuit board material, 1/16"-thick epoxy-reinforced fiberglass (FR4), with copper electrodes.
- the brass quadrants, as supplied with stainless-steel dowel pins in place, and as mounted on the copper rotor disc, describe arcs with an outer diameter of 3.72", an inner diameter of 1.72", and have a total mass of 214.5 ± 0.5 g each. The moment of inertia of a single quadrant is given by $\Delta I = \frac{1}{2}M(R_{\text{outer}}^2 + R_{\text{inner}}^2)$.
- The steel balls are chromium-steel bearing balls, of diameter, 1.0000", with amazingly tight tolerances on diameter and roundness. They have mass of 66.8 g each. Placed in the conical depressions atop either the copper rotor disc or the brass quadrants, their centers lie on a circle of diameter, $2.720" \pm 0.005"$.
- The rotor shaft is made of aluminum, and has maximal outer diameter of 1.50", typical outer diameter of 1.00", and a 0.38"-diameter hole through most of the length of its axis. The aluminum part, without magnets or mounting screws, has a mass of approximately 283 g.
- The magnets on the rotor shaft are nickel-plated NdFeB discs, each with a diameter of 1.00" and a thickness of 0.25". The stack of four magnets is separated at its center by a rib, 0.24" thick, that is a part of the rotor shaft. The mass of the four magnets together is 97 ± 1 g.
- The torsion fibers supplied with the apparatus are music wire conforming to ASTM A228, with very tight control over the diameter. The nominal diameters are 0.029", 0.039", 0.047", and 0.055", and these should be reliable in value, and in roundness, to $\pm 0.0005"$. The fibers are supplied with nominal lengths of 30", and the measured masses of the four fibers are 2.53 g, 4.62 g, 6.56 g, and 9.12 g respectively. The rotor disk is supported by *two* fibers (one above and one below) each of length 254 mm. The shear modulus G for ASTM A228 is approximately 80 GPa.² Unfortunately, we do not have a reliable estimate of the uncertainty in that value.
- Each 'air paddle' is made of a 20" piece of aluminum tubing, of outer diameter 0.250" and wall thickness of 0.014", and each tube has a measured mass of about 9.2 g. The paddle itself is constructed of foil-covered foam, nominally 6" by 4.5" in size, of a measured mass of 8.4 g. Of the tubing's length, 3" is immersed in the foam.

¹Adapted from section 6.1 of the TeachSpin Instruction Manual, Revision 0.9 10/28/08.

²see <http://www.matweb.com> and search for ASTM A228