Physics 238

Damped Oscillations in LRC Circuits
Due Monday, April 6, 2026

1 INTRODUCTION

In previous experiments, you observed various features of the damped and undamped linear harmonic oscillator, including the decay of free vibrations and the resonant response to external periodic driving.

Instead of a mechanical oscillator, this lab will use an electrical inductor-resistor-capacitor (LRC) circuit. You will use your results to determine the natural frequency f_0 and the quality factor Q. Along the way, you will gain experience using a digital oscilloscope to characterize time-varying signals.

2 THEORY

Throughout this writeup, we will adopt the convention of using lower case variables, e.g. v, for time varying quantities, and upper case variables, e.g. V, for quantities (such as amplitudes) that do not vary in time. Thus if the input voltage is sinusoidal, we would write it as

$$v_{\text{in}}(t) = V_{\text{in}} \sin(2\pi f t) \ .$$

Consider the circuit shown below in Fig. 1. The circuit is driven by an input voltage $v_{\text{in}}(t)$. We will measure the output voltage across the capacitor, so that $v_{\text{out}} = q/C$, where q is the charge on the capacitor and C is the capacitance. The current is given by $i = \dot{q}$, and the derivative of the current $\frac{di}{dt} = \ddot{q}$. Applying Kirchhoff's Voltage Law to this circuit yields a differential equation for the charge q:

$$\begin{aligned} v_{\text{in}}(t) - L\frac{di}{dt} - iR - \frac{q}{C} &= 0 \\ L\ddot{q} + R\dot{q} + \frac{q}{C} &= v_{\text{in}}(t) \\ \ddot{q} + \frac{R}{L}\dot{q} + \frac{1}{LC}q &= \frac{1}{L}v_{\text{in}}(t) \end{aligned} \tag{1}$$

This equation is analogous to that for the damped driven torsional oscillator

$$\begin{aligned} I\ddot{\theta} + b\dot{\theta} + \kappa\theta &= F_{\text{in}}(t) \\ \ddot{\theta} + \frac{b}{I}\dot{\theta} + \frac{\kappa}{I}\theta &= \frac{1}{I}F_{\text{in}}(t) \end{aligned} \tag{2}$$

where I is the moment of inertia, b is the viscous damping, and κ is the torsional constant.

Both equations are of the same generic form:

$$\ddot{q} + \gamma\dot{q} + \omega_0^2 q = a_{\text{in}}(t) \tag{3}$$

where $a_{\text{in}}(t)$ is proportional to the driving.

As in the torsional oscillator, there are several convenient terms:

$$\omega_0 = \frac{1}{\sqrt{LC}}$$
$$\gamma = \frac{R}{L}$$
$$Q = \frac{\omega_0}{\gamma}$$
$$f_0 = \frac{\omega_0}{2\pi}$$

f_0 is known as the natural frequency of the system—the frequency at which it would naturally oscillate in the absence of driving or forcing. Q is known as the quality factor. It is a dimensionless number that characterizes the system. Values of $Q \gg \frac{1}{2}$ indicate light damping.

If the system is set to an initial non-zero voltage and then allowed to relax, it will exhibit decaying oscillations, given by an equation of the form

$$v_{\text{out}}(t) = V_{\text{out}} e^{-\gamma t/2} \cos(2\pi f t - \phi) + V_{\text{off}} \tag{4}$$

where V_{off} accounts for any residual offset in the voltage readings, and should be very close to zero in this experiment. (You could use either sin or cos, but cos is a better match for the triggering in this experiment, and more convenient for the fits in §4.3.2 below.)

3 EXPERIMENT

3.1 Initial Setup

For this experiment, you will need an $L = 15\,\text{mH}$ inductor, an $R = 150\,\Omega$ resistor, and a $C = 22\,\text{nF}$ capacitor.[1] These are all nominal values. You should measure the actual values with a DMM and use those actual values in all your calculations. For the inductor, you will need to use the black "Kelvin" meter, and use the two thin slots labeled `Lx`. (The legs of the inductor are too short to fit in those slots, so you need to use a cable with small clamps.) Also, with the digital multimeter (DMM) set to measure resistance, measure the resistance of your inductor. For an ideal inductor it would be zero, but for a real inductor made out of a coil of thin wire, it will be a significant, measurable value. The manufacturer listed it as $37\,\Omega$, but you should measure and use the actual value. Similarly for the capacitor, you need to use the two terminals on the multimeter labeled `Cx`.

Construct a series LRC circuit on your breadboard, as shown in Figs. 1 and 2. Use the "Lo Ω" output of the PASCO function generator. (This output has an internal resistance of $8\,\Omega$, while the "Hi Ω" output has an internal resistance of $600\,\Omega$. That $600\,\Omega$ would be large enough to provide significant damping in this experiment.) If you use the banana-to-BNC adapter, make sure to put the side with the small nub labeled "GND" in the black "GND" terminal of the function generator.

[1] The capacitor will likely be labeled "223" for $22 \times 10^3\,\text{pF} = 22\,\text{nF} = 0.022\,\mu\text{F}$. It is a film capacitor. For more information, see the "Marking" section of the "ceramic capacitor" article in Wikipedia. Similarly, the inductor will likely be labeled "153" for $15 \times 10^3\,\mu\text{H} = 15\,000\,\mu\text{H} = 15\,\text{mH}$.

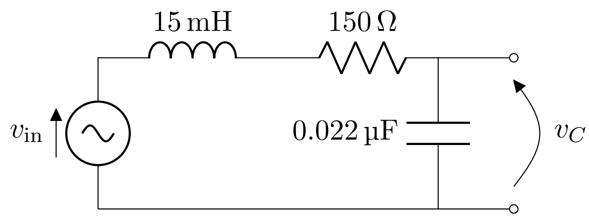

Figure 1: LRC series circuit.

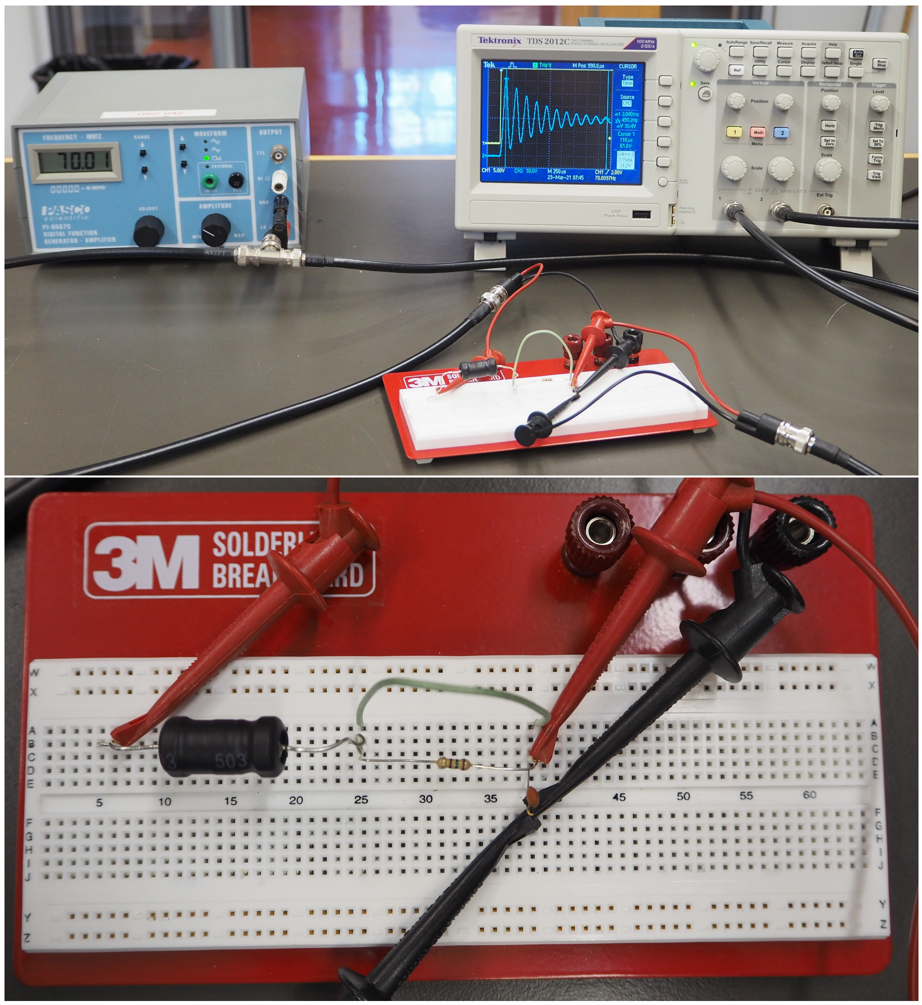

Figure 2: Photo of LRC series circuit. The green wire is the jumper wire across the 150 Ω resistor for the "undamped" oscillation measurement in §3.3.

When setting up electrical circuits, it usually pays to set things up carefully so that the wires don't get confused. It is often useful to set up the physical circuit to match the schematic diagram, as in Figs. 1 and 2.

Turn the function generator and oscilloscope on. Set the function generator to produce a sine wave of about 1 kHz (this is the default power-on setting). Turn the amplitude knob about 1/4 of the way up. Use Channel 1 of the oscilloscope to measure the *input* voltage, and Channel 2 to measure the *output* voltage v_C across the capacitor. Since $v_C = q/C$, Ch. 2 is (effectively) measuring the charge. (You will have to use a "T" adapter to run two cables from the function generator—one to the circuit and the other to the oscilloscope.)

You should see the resulting sine wave on the oscilloscope. Be sure that both inputs on the oscilloscope are set to the "1X" setting. You want the input wave to have a peak-to-peak amplitude of no more than 2 V.

If you are not familiar with this oscilloscope, now is a good time to take a few minutes to experiment with the buttons on the front to see how it works. Feel free to consult the User Manual that is kept in the lab. (The manual is either on the front table, or on the bookshelf on the right-hand side of the back wall.) The `Auto Set` button is often helpful for initially finding a signal. You will likely find that the `Cursor` is particularly useful for the next three parts, while the automatic `Measure` option will be a great time-saver when mapping out the resonance curve in the next experiment.

3.2 Acquiring and Saving Data

There are a couple of options for downloading data and screenshots from the oscilloscope. You can use a USB drive to save data and images. See the instruction manual under "Data Logging."

The oscilloscope can also be connected to the computer via a USB cable. Launch the `OpenChoiceDesktop` program. This program can

- Capture screen images and save as PNG files, suitable for import into LaTeX.
- Save raw Ch1 and Ch2 data (you need to select Ch 2 on the channels menu) as a "Comma Separated Values" (csv) file, suitable for import *Mathematica*.[2]

3.3 Undamped Oscillations

The first task is to measure f_0, the frequency of *undamped* oscillations. For this circuit, that means we want the resistance R to be zero. Use a jumper wire to bypass (or short out) the resistor. (This is the green wire in Fig. 2.) We can not avoid the internal resistance of the function generator and of the coils in the inductor, but we can hope (for the moment) that those resistances are small.

Set the function generator to a frequency of about 70 Hz and set the output to square waves. This will give the circuit repeated "kicks," after which it will oscillate at its natural frequency.

Use the `Cursor` menu to time one or more complete oscillations and calculate the natural frequency f_0 of oscillation. (Note that if you use `Cursor 1` and `Cursor 2` to measure the times for the peaks of successive oscillations, the oscilloscope calculates the frequency for you automatically.)[3] Call this frequency f_0. *Be sure to spread out the horizontal and vertical scales so that you get a good clean measurement, as in Fig. 3. You may also have to fiddle with the* `Triggering` *control to get a stable display. Setting the Triggering*

[2]The raw data from the oscilloscope contains a lot of extra information. You may find it helpful to load the file up in a spreadsheet to explore the data and decide which parts you need to use to import the relevant Ch. 2 data.

[3]See the example "Measuring Ring Frequency" on pg. 48 of the User Manual for step-by-step instructions.

to a falling *slope is usually helpful. You may either zoom in on one oscillation cycle, or zoom out and measure the total time for a larger number of cycles. Feel free to ask your instructor for help.*

Hint: You can't reliably determine the start of the first cycle; start your measurements after the oscillatory pattern is clearly established.

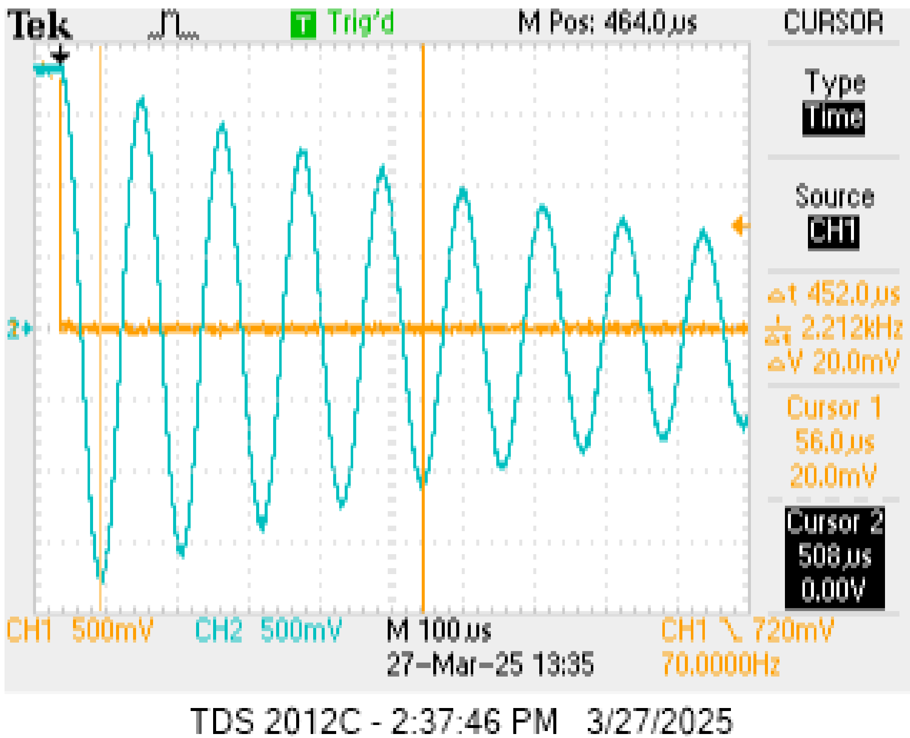

Figure 3: Measuring the frequency of the undamped oscillations. This figure shows the cursors separated by 4 full cycles, so $4T_0 = 452.0\,\mu\text{s}$.

Try making small adjustments to the cursor positions. What do you estimate is your uncertainty in f_0? Call that δf_0. (This is not a statistical analysis here — just an estimate of the uncertainty.)

Save a screenshot of your "undamped" oscillation. Also, save the raw data as a `.csv` file. Be sure to include both channels 1 and 2.

3.4 Damped Oscillations

Remove the jumper wire that was shorting out the resistor. You have now introduced significant damping into the system. Measure the new frequency of oscillation. This is the *damped* frequency f_v of your oscillator. (The difference from f_0 should be quite small.) Again, estimate δf_v as well.

Save a screenshot of your "damped" oscillation. Also, save the raw data as a `.csv` file. Be sure to include both channels 1 and 2.

3.5 The Decay of Free Vibrations

If necessary, adjust the vertical position of the trace on the oscilloscope so that the voltage is oscillating about a value of 0. This works best if you set the trigger to trigger on a *Falling* slope, rather than the default *Rising* slope.

Use the `Cursors` to measure the height (voltage) of each peak and trough. One convenient way to do that is to set `Cursor 1` to the average voltage of the signal after it decays, and use `Cursor 2` to measure each peak and trough. Note that the oscilloscope will report ΔV, which is what you want to record. Measure as many peaks and troughs as you can with reasonable accuracy. Include the sign.

You can't reliably measure the initial peak. If you signal looks similar to Fig. 3, then the first thing you can measure is a trough. Call that peak $n = 0.5$. Since it is a trough, that voltage would be negative.

4 ANALYSIS

4.1 Expected Values

Use your measured values of R, L, and C to predict values for f_0 and Q.

4.2 Undamped Oscillations

4.2.1 Oscilloscope Measurements

Use your measured value for the period of oscillation from the oscilloscope screen to determine f_0 and its uncertainty.

4.2.2 Curve Fit Measurements

Fit your "undamped" data to a function of the form of Eq. 4. (Since there is resistance in the inductor, there is still some damping, but this is as close as we can get to "undamped.") The raw data from the oscilloscope contains a lot of extra information. You may find it helpful to load the file up in a spreadsheet to explore the data and decide which parts you need to use to import the relevant Ch. 2 data.

What value do you get for f_0 from this fit? Again, include the uncertainty as well.

4.3 Damped Oscillations

4.3.1 Oscilloscope Measurements

Use your measured value for the period of oscillation from the oscilloscope screen to determine f_v and its uncertainty. This is the frequency of the *damped* oscillations. Recall that the damped and undamped frequencies are related to the quality factor Q by

$$f_v = f_0\sqrt{1-\frac{1}{4Q^2}}\,. \tag{5}$$

Can you use this to estimate the quality factor Q? Note that both f_0 and f_v have uncertainty. Is your precision in them high enough to warrant doing this calculation? If so, give your result for Q. You may omit any calculation of δQ. In practice, the value obtained for Q this way is rarely very reliable in this experiment.

4.3.2 Decay of Free Vibration Peaks

Next, you will determine Q from the decaying amplitude of the vibration peaks. Recall that the motion is predicted to be a decaying oscillation of the form Eq. 4.

However, instead of $v(t)$, your data is in the form $v(n)$ *vs.* n, where n is the oscillation number, and $v(n)$ is the amplitude of the voltage across the capacitor at the n^{th} oscillation. (Since you measured both peaks and troughs, you actually have $n = 0$, $n = \frac{1}{2}$, $n = 1$, $n = \frac{3}{2}$, *etc.*)

Rewrite Eq. 4 to be in terms of n and Q. Specifically, rewrite γ in terms of f_0 and Q. Then assume that $f_v = f_0$, and that your measurements occurred at times given by $t = nT_0$ (where $T_0 = 1/f_0$).

Fit your revised equation to the peak data to find Q, and its uncertainty.

4.3.3 Curve Fit Measurements

Fit your "damped" data to a function of the form of Eq. 4. The frequency you get here is the "damped" frequency f_v. What value do you get for f_v from this fit? Repeat the analysis using Eq. 5 to estimate Q from this value, if the data warrant doing the calculation.

Use the fit values for γ and f_0 to determine the value for

$$Q = \frac{\omega_0}{\gamma} = \frac{2\pi f_0}{\gamma}\,.$$

4.4 Overall Comparisons

In this lab, you found f_0 in multiple different ways: from the measured values of the components, from the frequency of the undamped oscillator as measured on the oscilloscope, and from the curve fit to the undamped oscillation. Compare your values and comment on any notable similarities or differences.

You also calculated Q in four different ways: from the measured values of the components, from the frequencies of damped and undamped oscillations, from the peak measurements of the decay of free vibrations, and from the fit to the damped oscillation curve. Compare your values and comment on any notable similarities or differences.

There is a wealth of data here; be sure to take the time and care to present it clearly and carefully.