Physics 238

Resonance in LRC Circuits
Due Monday, April 13, 2026

1 INTRODUCTION

In previous experiments, you observed various features of the damped and undamped linear harmonic oscillator, including the decay of free vibrations and the resonant response to external periodic driving.

In this lab, you will study resonance for an electrical inductor-resistor-capacitor (LRC) circuit. You will use your results to determine the natural frequency f_0 and the quality factor Q. Along the way, you will gain more experience using a digital oscilloscope to characterize time-varying signals.

2 THEORY

Throughout this writeup, we will adopt the convention of using lower case variables, e.g. v, for time varying quantities, and upper case variables, e.g. V, for quantities (such as amplitudes) that do not vary in time. Thus if the input voltage is sinusoidal, we would write it as

$$v_{\text{in}}(t) = V_{\text{in}} \sin(2\pi f t) \; .$$

Consider the circuit shown below in Fig. 1. The circuit is driven by an input voltage $v_{\text{in}}(t)$. We will measure the output voltage across the capacitor, so that $v_{\text{out}} = q/C$, where q is the charge on the capacitor and C is the capacitance. The current is given by $i = \dot{q}$, and the derivative of the current $\frac{di}{dt} = \ddot{q}$. Applying Kirchhoff's Voltage Law to this circuit yields a differential equation for the charge q:

$$\begin{aligned} v_{\text{in}}(t) - L\frac{di}{dt} - iR - \frac{q}{C} &= 0 \\ L\ddot{q} + R\dot{q} + \frac{q}{C} &= v_{\text{in}}(t) \\ \ddot{q} + \frac{R}{L}\dot{q} + \frac{1}{LC}q &= \frac{1}{L}v_{\text{in}}(t) \end{aligned} \tag{1}$$

This equation is analogous to that for the damped driven torsional oscillator

$$\begin{aligned} I\ddot{\theta} + b\dot{\theta} + \kappa\theta &= F_{\text{in}}(t) \\ \ddot{\theta} + \frac{b}{I}\dot{\theta} + \frac{\kappa}{I}\theta &= \frac{1}{I}F_{\text{in}}(t) \end{aligned} \tag{2}$$

where I is the moment of inertia, b is the viscous damping, and κ is the torsional constant.

Both equations are of the same generic form:

$$\ddot{q} + \gamma\dot{q} + \omega_0^2 q = a_{\text{in}}(t) \tag{3}$$

where $a_{\text{in}}(t)$ is proportional to the driving.

As in the torsional oscillator, there are several convenient terms:

$$\omega_0 = \frac{1}{\sqrt{LC}}$$
$$\gamma = \frac{R}{L}$$
$$Q = \frac{\omega_0}{\gamma}$$
$$f_0 = \frac{\omega_0}{2\pi}$$

f_0 is known as the natural frequency of the system—the frequency at which it would naturally oscillate in the absence of driving or forcing. Q is known as the quality factor. It is a dimensionless number that characterizes the system. Values of $Q \gg \frac{1}{2}$ indicate light damping.

If the system is set to an initial non-zero voltage and then allowed to relax, it will exhibit decaying oscillations, given by an equation of the form

$$v_{\text{out}}(t) = V_{\text{out}} e^{-\gamma t/2} \cos(2\pi f_v t - \phi) + V_{\text{off}} \tag{4}$$

where V_{off} accounts for any residual offset in the voltage readings, and should be very close to zero in this experiment. (You could use either sin or cos, but cos is a better match for the triggering in this experiment.)

If instead the system is driven by a sinusoidal voltage with amplitude V_{in} and frequency f_d

$$v_{\text{in}} = V_{\text{in}} \cos(2\pi f_d t) \tag{5}$$

then the voltage across the capacitor will (after a transient) oscillate at the same frequency, but with a different amplitude V_C and phase ϕ

$$v_{\text{c}} = V_C \cos(2\pi f_d t - \phi) \tag{6}$$

where the amplitude V_C is given by

$$V_C(\omega_d) = \frac{V_{\text{in}}/LC}{\sqrt{(\omega_0^2 - \omega_d^2)^2 + (\omega_d \omega_0/Q)^2}} \quad \text{, or}$$
$$\frac{V_C(f_d)}{V_{\text{in}}} = \frac{f_0^2}{\sqrt{(f_0^2 - f_d^2)^2 + \left(\frac{f_d f_0}{Q}\right)^2}} \tag{7}$$

where the second line follows from noting that $\omega_0^2 = 1/LC$, and using $\omega = 2\pi f$ throughout. The phase ϕ is given by

$$\tan\phi = \frac{\gamma \omega_d}{\omega_0^2 - \omega_d^2} = \frac{f_0 f_d}{Q(f_0^2 - f_d^2)} \tag{8}$$

3 EXPERIMENT

3.1 Initial Setup

This experiment uses the same components as the LRC damping experiment; if everything is still set up from that experiment, you can re-use your previous values and data for §3.3 and skip ahead to §3.4.

For this experiment, you will need an $L = 15\,\text{mH}$ inductor, an $R = 150\,\Omega$ resistor, and a $C = 22\,\text{nF}$ capacitor.[1] These are all nominal values. You should measure the actual values with a DMM and use those actual values in all your calculations. For the inductor, you will need to use the black "Kelvin" meter, and use the two thin slots labeled `Lx`. (The legs of the inductor are too short to fit in those slots, so you need to use a cable with small clamps.) Also, with the digital multimeter (DMM) set to measure resistance, measure the resistance of your inductor. For an ideal inductor it would be zero, but for a real inductor made out of a coil of thin wire, it will be a significant, measurable value. The manufacturer listed it as $37\,\Omega$, but you should measure and use the actual value. Similarly for the capacitor, you need to use the two terminals on the multimeter labeled `Cx`.

Construct a series LRC circuit on your breadboard, as shown in Figs. 1 and 2. Use the "Lo Ω" output of the PASCO function generator. (This output has an internal resistance of $8\,\Omega$, while the "Hi Ω" output has an internal resistance of $600\,\Omega$. That $600\,\Omega$ would be large enough to provide significant damping in this experiment.) If you use the banana-to-BNC adapter, make sure to put the side with the small nub labeled "GND" in the black "GND" terminal of the function generator.

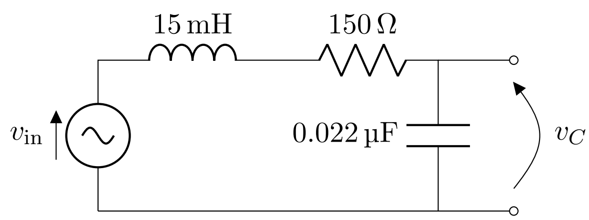

Figure 1: LRC series circuit.

When setting up electrical circuits, it usually pays to set things up carefully so that the wires don't get confused. It is often useful to set up the physical circuit to match the schematic diagram, as in Figs. 1 and 2.

Turn the function generator and oscilloscope on. Set the function generator to produce a sine wave of about 1 kHz (this is the default power-on setting). Turn the amplitude knob about 1/4 of the way up. Use Channel 1 of the oscilloscope to measure the *input* voltage, and Channel 2 to measure the *output* voltage v_C across the capacitor. Since $v_C = q/C$, Ch. 2 is (effectively) measuring the charge. (You will have to use a "T" adapter to run two cables from the function generator—one to the circuit and the other to the oscilloscope.)

You should see the resulting sine wave on the oscilloscope. Be sure that both inputs on the

[1] The capacitor will likely be labeled "223" for $22 \times 10^3\,\text{pF} = 22\,\text{nF} = 0.022\,\mu\text{F}$. It is a film capacitor. For more information, see the "Marking" section of the "ceramic capacitor" article in Wikipedia. Similarly, the inductor will likely be labeled "153" for $15 \times 10^3\,\mu\text{H} = 15\,000\,\mu\text{H} = 15\,\text{mH}$.

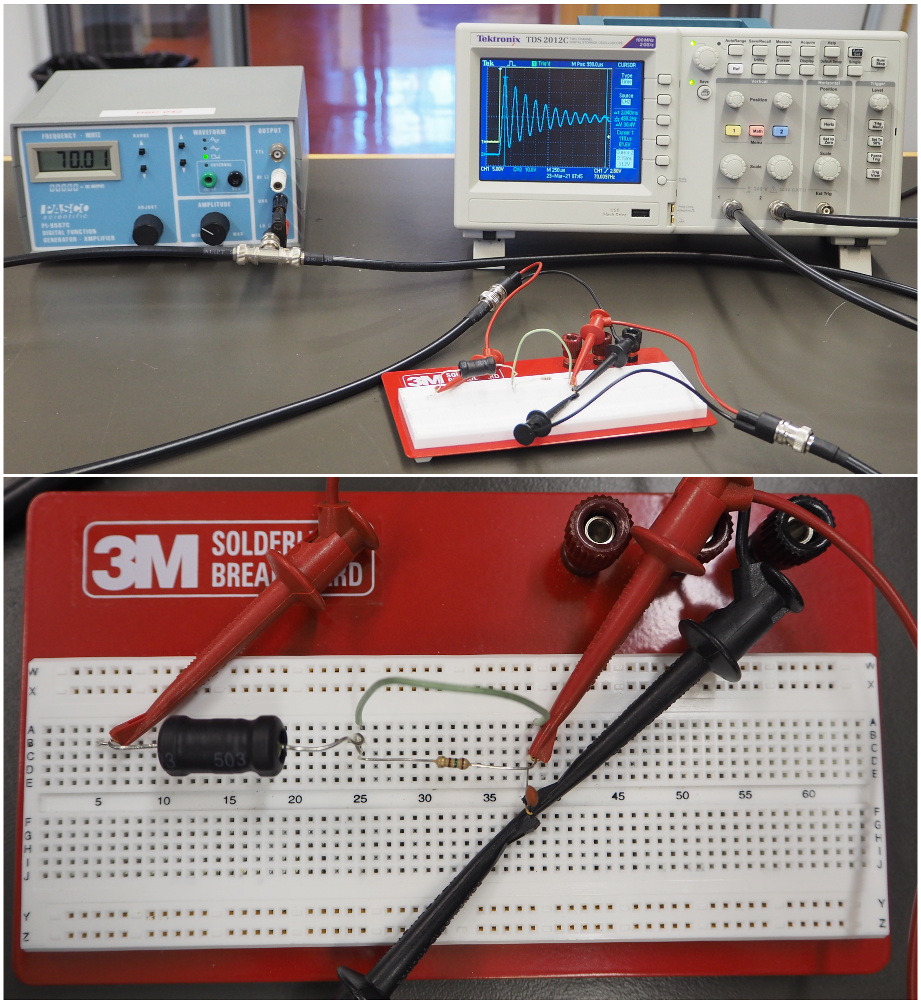

Figure 2: Photo of LRC series circuit. The green wire is the jumper wire across the 150 Ω resistor for the "undamped" oscillation measurement in the previous experiment.

oscilloscope are set to the "1X" setting. You want the input wave to have a peak-to-peak amplitude of no more than 2 V.

If you are not familiar with this oscilloscope, now is a good time to take a few minutes to experiment with the buttons on the front to see how it works. Feel free to consult the User Manual that is kept in the lab. (The manual is either on the front table, or on the bookshelf on the right-hand side of the back wall.) The `Auto Set` button is often helpful for initially finding a signal. You will likely find that the `Cursor` is particularly useful for the next three parts, while the automatic `Measure` option will be a great time-saver when mapping out the resonance curve in the next experiment.

3.2 Acquiring and Saving Data

There are a couple of options for downloading data and screenshots from the oscilloscope. You can use a USB drive to save data and images. See the instruction manual under "Data Logging."

The oscilloscope can also be connected to the computer via a USB cable. Launch the `OpenChoiceDesktop` program. This program can

- Capture screen images and save as PNG files, suitable for import into LaTeX.
- Save raw Ch. 1 and Ch. 2 data (you need to select Ch. 2 on the channels menu) as a "Comma Separated Values" (csv) file, suitable for import *Mathematica*.[2]

3.3 Damped Oscillations

The first task is to estimate f_0, the frequency of *undamped* oscillations, and Q. Set the function generator to a frequency of about 70 Hz and set the output to square waves. This will give the circuit repeated "kicks," after which it will oscillate approximately at its natural frequency. Recall that for this system, the frequencies of the damped f_v and undamped f_0 oscillator were nearly indistinguishable. So you will measure f_v, but use that value as an estimate for f_0.

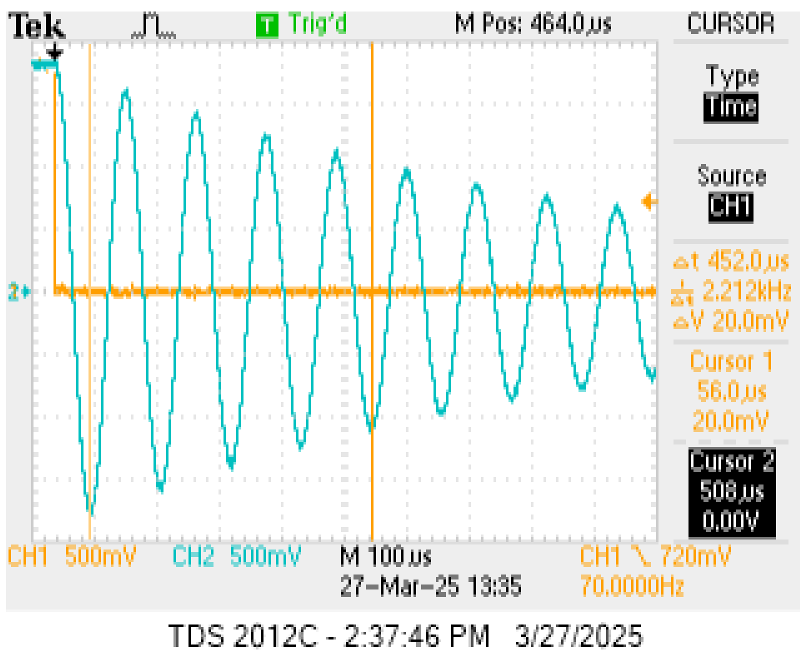

Figure 3: Measuring the frequency of the undamped oscillations. This figure shows the cursors separated by 4 full cycles, so $4T_0 = 452.0\,\mu\text{s}$.

Use the `Cursor` menu to time one or more complete oscillations and calculate the frequency of oscillation. Call this frequency f_0. *Be sure to spread out the horizontal and vertical scales so that you get a good clean measurement, as in Fig. 3. You may also have to fiddle with the* ***`Triggering`*** *control to get a stable display. Setting the Triggering to a* falling *slope is*

[2] The raw data from the oscilloscope contains a lot of extra information. You may find it helpful to load the file up in a spreadsheet to explore the data and decide which parts you need to use to import the relevant Ch. 2 data.

usually helpful. You may either zoom in on one oscillation cycle, or zoom out and measure the total time for a larger number of cycles. Feel free to ask your instructor for help.

Hint: You can't reliably determine the start of the first cycle; start your measurements after the oscillatory pattern is clearly established.

Save the raw data for your damped oscillation as a `.csv` file. Be sure to include both channels 1 and 2.

3.4 Forced Oscillations and Resonance

The main goal here is to make a series of measurements to determine how the steady-state voltage amplitude and phase depend on frequency.

3.4.1 Preliminary Calculations

It is useful to plan ahead. Use your current best estimates for f_0 and Q (either from the damped oscillations in §3.3 or from a previous experiment) to make a theoretical plot of Eq. 7. Plan a set of approximately 20 measurements that will be useful to measure that curve. You should take a few measurements at frequencies far from the resonance peak, and more measurements closer to the peak. The goal is to be able to measure the full shape of the peak effectively but efficiently.

3.4.2 Data Acquisition

Set the function generator to sine waves. Set the oscilloscope to `Measure` and have it automatically measure the Cyclic-RMS voltages for Channels 1 and 2. (The RMS voltage is much less sensitive to noise fluctuations than the peak-to-peak voltage.) Also set the oscilloscope to measure the phase difference between Ch 2 and Ch 1. (The Oscilloscope will report that phase in degrees. You will later need to convert that to radians.) You will record the frequency directly from the function generator.

Next, try to find a reasonable amplitude for the driving signal. Set the function generator to a frequency close to f_0. Adjust the amplitude on the function generator so that you get a good signal. A peak amplitude of about 500 mV for Ch. 1 often works well. You may also have to adjust the vertical "Volts per Division" knobs on the oscilloscope. Your goal here is to be sure you have a reasonable amplitude and that the signals remain sinusoidal. Keeping the output peak amplitude less than about 2 V for Ch. 2 usually works well. If needed, you may adjust the amplitude on the function generator, as well as the vertical "Volts per Division" knobs on the oscilloscope as you change frequency. You should also change the horizontal scale. The goal is to always have a good clean sinusoidal signal that can be measured well.

Now make a series of measurements to determine how the steady-state voltage amplitude depends on frequency. Specifically, record a table with the following for each frequency:

$$f \quad V_{\text{in}} \quad V_{\text{out}} \quad \Delta\phi$$

where f can be read from the function generator, V_{in} and V_{out} are the input and output RMS voltages, and $\Delta\phi$ is the phase difference between the Ch. 2 and Ch. 1 voltage peaks.

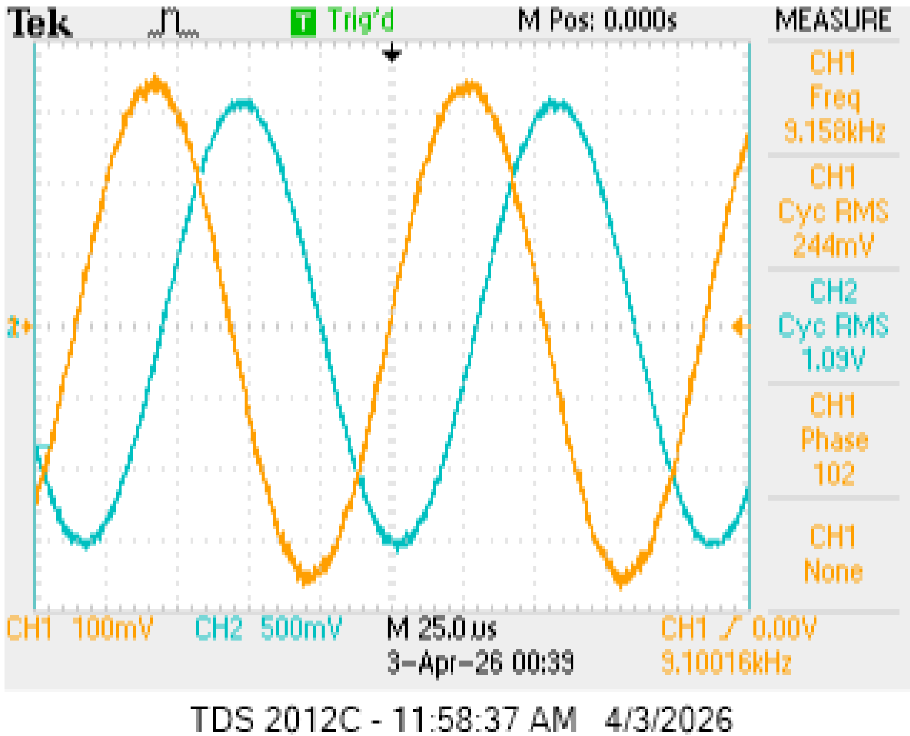

Figure 4: Measuring the RMS amplitudes and phase difference of the input and output voltages. Note that Ch. 1 and Ch. 2 have different scales, and the horizontal scale knob has been adjusted to give at least one complete cycle on screen.

You should take enough data points to get a smooth curve, but don't waste time taking too much redundant data. Adjust the horizontal and vertical scale knobs on the oscilloscope to make sure several complete oscillation cycles are easily visible, and blown up to fill most of the screen. Make sure to take a few measurements at high and low frequency. Note that on the PASCO generators, there are push buttons to change the frequency *range*. The extreme frequencies should be far enough from resonance that the amplitude is much smaller than the amplitude at resonance. *If you plot your data as you go along, it will usually be clear when you have enough data.*

4 ANALYSIS

4.1 Expected Values

Use your measured values of R, L, and C to predict values for f_0 and Q.

4.2 Damped Oscillations

4.2.1 Curve Fit Measurements

Fit your "damped" data to a function of the form of Eq. 4. If your set-up is unchanged from the previous experiment, you may simply copy that result here. The frequency you get here is the "damped" frequency f_v. What value do you get for f_v from this fit? Recall that for this experiment, this is also a reasonable estimate for the *undamped* frequency f_0.

- Is Eq. 4 a good *qualitative* fit to your data?
- Include a graph showing your data along with the best fit. Be sure to give appropriate axes labels, and to make the graph readable.
- Report your best fit values for Q and f_0, along with their uncertainties.

4.3 Forced Oscillations and Resonance

4.3.1 Amplitude

Perform a fit of Eq. 7 to your data for the steady-state gain $\frac{V_{\text{out}}}{V_{\text{in}}}$ *vs.* frequency. Let *Mathematica* vary Q and f_0. Include a graph showing your data along with the fit in your report. Be sure to give appropriate axes labels, and to make the graph readable.

- Is Eq. 7 a good *qualitative* fit to your data?
- Include a graph showing your data along with the best fit. Be sure to give appropriate axes labels, and to make the graph readable.
- Report your best fit values for Q and f_0, along with their uncertainties.

4.3.2 Phase

Fit Eq. 8 to your data for phase *vs.* frequency. Note that in order to get the correct quadrant from *Mathematica*, you need to use the two-argument form of `ArcTan[x, y]`.

- Is Eq. 8 a good *qualitative* fit to your data?
- Include a graph showing your data along with the best fit. Be sure to give appropriate axes labels, and to make the graph readable.
- Report your best fit values for Q and f_0, along with their uncertainties.

4.4 Overall Comparisons

In this lab, you found f_0 in four different ways: from the measured values of the components, from the frequency of the damped oscillator, and from the curve fits to Eqs. 7 and 8. Compare your values and comment on any notable similarities or differences.

Similarly, you also calculated Q in four different ways. Compare your values and comment on any notable similarities or differences.

There is a wealth of data here; be sure to take the time and care to present it clearly and carefully. Presenting the results in neat tables is often helpful. Comment on any significant trends or findings. Discuss any relevant fit diagnostics.