Physics 238

Optical Spectroscopy II: Hydrogen and Deuterium

Report **Due May 1, 2026**

1 Hydrogen Spectrum and the Balmer Series

1.1 Introduction

When an electric discharge passes through a gas of neutral atoms, all of the same chemical element, spectral lines (each consisting of light of a specific wavelength) are produced. These lines can tell us a great deal about the internal structure of the atom, since they are produced when electrons in the atoms make transitions from one energy level to another.

The spectrum of wavelengths emitted by hydrogen has an especially simple regularity that was found empirically to be given by:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n_l^2} - \frac{1}{n_u^2} \right) \tag{1}$$

where R_H a constant known as the Rydberg constant (for Hydrogen), and n_l and n_u are integers. One of the great triumphs of Bohr's model of hydrogen was its ability to reproduce the observed spectrum of hydrogen and provide a first-principles value for the Rydberg constant.

In this experiment, you will use the SPEX 1250M spectrometer to measure the spectrum of light from hydrogen and deuterium. You will measure the wavelengths of the four longest-wavelength lines from the Balmer series of hydrogen and deuterium ($n_l = 2$) and determine the mass of the deuterium nucleus.

1.2 Theory for the Hydrogen and Deuterium Spectrum

The energy levels for an electron in a hydrogen atom are given by

$$E_n = -\frac{m_e e^4}{(4\pi\epsilon_0)^2\, 2\hbar^2} \frac{1}{n^2},$$

where m_e and e are the mass and charge of an electron, ϵ_0 is the permittivity of free space, $\hbar$ is Planck's constant, and $n = 1, 2, 3...$ is the quantum number denoting the energy level, and where it is assumed that the nucleus is infinitely massive (so that it does not move). An atom making a transition from an upper state n_u to a lower state n_l loses energy ΔE,

$$\Delta E = E_u - E_l = \frac{m_e e^4}{(4\pi\epsilon_0)^2 \, 2\hbar^2} \left(\frac{1}{n_l^2} - \frac{1}{n_u^2} \right) .$$

The atom loses energy ΔE by emitting a photon of wavelength λ:

$$\Delta E = \frac{hc}{\lambda} = \frac{2\pi\hbar c}{\lambda}$$

$$\frac{1}{\lambda} = \frac{\Delta E}{2\pi\hbar c} = \frac{m_e e^4}{(4\pi\epsilon_0)^2 \, 4\pi\hbar^3 c} \left(\frac{1}{n_l^2} - \frac{1}{n_u^2} \right) ,$$

where c is the speed of light.

This can be written as

$$\frac{1}{\lambda} = R_\infty \left(\frac{1}{n_l^2} - \frac{1}{n_u^2} \right) \tag{2}$$

where R_∞ is called the Rydberg constant, and is equal to

$$R_\infty = \frac{m_e e^4}{(4\pi\epsilon_0)^2 \, 4\pi\hbar^3 c} = 1.097\,373 \times 10^7 \,\mathrm{m}^{-1} .$$

This value would be appropriate if the nucleus were infinitely massive. For a nucleus of finite mass m_N, the reduced mass μ_N should be used instead of the electron mass m_e:

$$\mu_N = \frac{m_N m_e}{m_N + m_e} \tag{3}$$

With the appropriate reduced mass (μ_H or μ_D), you obtain different values of the Rydberg for hydrogen (R_H) and for deuterium (R_D).

When $n_l = 2$, the *Balmer* series is produced. The only visible Balmer lines occur when n_u = 3, 4, 5, and 6; the remainder are in the ultraviolet range.

In this experiment, you will measure the wavelengths of 4 lines from the Balmer series of hydrogen and deuterium ($n_l = 2$) and use the differences in those wavelengths to determine the mass of the deuterium nucleus.

1.3 Experiment

The deuterium lamp should be set up at the front ("Axial") entrance slit of the SPEX 1250M spectrometer. Align the source carefully with the input. An input slit width of 5 µm and height of 0.2 cm usually works well.

Launch the `SynerJY` program. Use the `Monos` button to select the "Axial" input. Use the `Detectors` button to set the voltage to 850 V. The deuterium lamp tends to flicker, so setting a slightly longer integration time, such as 0.2 s or 0.4 s, usually works better.

The lamp should not be turned on for long periods of time. Turn off the lamp whenever you will not be taking measurements for at least 5 minutes or so. The source actually contains a significant amount of hydrogen in addition to the deuterium, so you should actually see a two peaks—one for deuterium and one for hydrogen.

Record a spectrum for each of the first four lines of the Balmer series. (The tube is made from glass that blocks ultraviolet light, so you will get the strongest signals from the first three or four lines which are in the visible range.) You will probably find it useful to pre-calculate where the peak ought to be, and then do a quick scan in that vicinity to be sure you have the appropriate range. A resolution of 0.001 nm works well for these final high-resolution scans. Export your data for each peak.

Record a spectrum for each of the first four lines of the Balmer series. You will probably find it useful to pre-calculate where the peak ought to be, and then do a quick scan in that vicinity to be sure you have the appropriate range. A resolution of 0.005 nm useful for the quick scans, while 0.001 nm is sufficient for your final high-resolution scans.

Make sure you have two clear peaks within each scan. Export your data for later use.

Lastly also do a very-high resolution scan 0.000 25 nm of the deuterium and hydrogen red lines and export that data.

2 Analysis

2.1 Deuterium

Measurements

1. For each scan, fit a sum of two gaussian peaks to your data. (No calibration adjustment is necessary since you will just be interested in the difference between the two wavelengths.).

2. For each of the four scans, compute the difference in the peak wavelengths, $\Delta\lambda = \lambda_H - \lambda_D$..

3. For the red line, $n_u = 3$, find an equation relating $\Delta\lambda$ to the deuteron mass m_D.

   - First, to be concrete, consider $n_u = 3$. It will be easy to generalize for $n_u = 4,\ 5,\ 6$.

- Use the Balmer formula to write an equation for $\Delta\lambda$.
- Solve your equation for the deuterium mass m_D. Assume you know the hydrogen mass m_H, as well as m_e and R_∞. The algebra can get messy. You can simplify things a bit by replacing R_H and R_D by

$$R_H = R_\infty \left(\frac{\mu_H}{m_e}\right) = R_\infty \left(\frac{m_H}{m_H + m_e}\right)$$
$$R_D = R_\infty \left(\frac{\mu_D}{m_e}\right) = R_\infty \left(\frac{m_D}{m_D + m_e}\right)$$

- Your equation will still be messy. It does simplify some, but you can also use a tool such as Mathematica to solve it numerically.

4. Repeat that analysis for $n_u = 4$, 5, and 6.

5. Use those four measurements to calculate a mean value for the mass of the deuteron. Calculate the uncertainty, *i.e.* the standard deviation of the mean, as well.

6. Compare your value with the expected value for the deuteron mass.

7. In your high-resolution scan for the deuterium α line, you should notice that the deuterium peak seems to be split by an amount $\Delta\lambda_{obs}$. That is an example of *fine structure*, which refers to the splitting of a spectral peak into two or more closely-spaced peaks. This particular splitting is due to spin-orbit coupling.

   However, you should also notice that the hydrogen line does *not* appear to be split. The likely explanation for this is Doppler broadening. In a hot gas in thermal equilibrium, molecules have a mean speed that depends on temperature. You can make a simple estimate of this effect in the following way:

   (a) Measure the splitting $\Delta\lambda_{obs}$ of the two peaks for deuterium. No peak-fitting is required here; just look at the graph and the data table and make your best estimate. The peak may be noisy, in which case you will have to make your best estimate.

   (b) Since you can resolve two peaks for deuterium, the Doppler effect for deuterium must be small enough to not obscure the separate peaks. Since deuterium atoms in the gas have a range of velocity components along the line of sight, you observed a corresponding range of wavelengths $\Delta\lambda_{dopp,D}$, where the subscript D means "deuterium". Measuring the peak shape and peak widths carefully is beyond the scope of this experiment. However, since you did observe the spin-orbit splitting, you may make the reasonable assumption that $\Delta\lambda_{dopp,D} < \Delta\lambda_{obs}$. As a reasonable estimate, assume

$$\Delta\lambda_{dopp,D} = \frac{1}{2}\Delta\lambda_{obs} \ .$$

(c) Assume that the speed of an atom along the line of sight is either $+v$ or $-v$, where

$$v = \sqrt{\frac{kT}{m}}$$

where k is Boltzmann's constant, T is the temperature, and m is the mass of the atom (just use the nuclear mass – ignore the electron). Also, assume that $\Delta\lambda_{dopp,D}$ is given by the difference between the Doppler-shifted wavelengths for $\pm v$. Using those assumptions, calculate $\beta = v/c$, and from that, calculate the quantity kT.

(d) Now, given that value for kT, calculate the expected peak width due to Doppler broadening $\Delta\lambda_{dopp,H}$ for hydrogen.

(e) Assume that the spin-orbit splitting $\Delta\lambda_{obs}$ is the same for hydrogen as it is for deuterium. Would you expect to be able to resolve the splitting of the hydrogen peak? Does this match your observations?