Mechanical Resonance — relating voltages to physical quantities

Driving

$$\tau = -\kappa\,\theta$$

$$\frac{d\tau}{dV} \cdot V_{in} = -\kappa \frac{d\theta}{dV} v_{out}$$

$$V_{in} = -\left(\kappa \frac{d\theta/dV}{d\tau/dV}\right) v_{out}$$

$$\ddot{\theta} = -\omega_0^2\,\theta - \frac{\omega_0}{Q}\dot{\theta} + \alpha_0 \sin \omega_d t$$

$$\ddot{V}_{out} = -\omega_0^2 V_{out} - \frac{\omega_0}{Q}\dot{V}_{out} + \frac{\tau_0}{I} \cdot \frac{1}{d\theta/dV} \sin \omega_d t$$

$$\ddot{V}_{out} = -\omega_0^2 V_{out} - \frac{\omega_0}{Q}\dot{V}_{out} + \left(\frac{d\tau/dV}{I\, d\theta/dV} V_0\right) \sin \omega_d t$$

Ultimately, we have

$$v_{in}(t) = V_{in} \sin(2\pi f_d t - \phi_{in})$$

$$v_{out}(t) = V_{out} \sin(2\pi f_d t - \phi_{out})$$

$$\frac{V_{out}}{V_{in}} = \frac{C}{\sqrt{(\omega_0^2 - \omega_d^2)^2 + (\gamma\,\omega_d)^2}}$$

"C" incorporates the various proportionality constants. Rewrite this in terms of f_d, f_0, and Q.

still have $\delta \equiv \phi_{out} - \phi_{in}$

$$\tan \delta = \frac{\gamma\,\omega_d}{\omega_0^2 - \omega_d^2}$$