Physics 238

The Torsional Oscillator
Part 2: Damped Oscillations
Report: **Due Wednesday, March 4, 2026**

Safety

This experiment involves permanent magnets with strong, but localized, magnetic fields. Just outside the wooden frame of the apparatus, the field is approximately 5 mT (*i.e.* 50 Gauss). This may be large enough to interfere with some sensitive medical devices. If you have any concerns, please consult your lab instructor.

1 INTRODUCTION

In the first part of this lab, you performed basic measurements with the torsional oscillator apparatus. Along the way, you determined the torsional spring constant κ and the moment of inertia. You also determined calibration factors for the output voltage as a function of angular displacement.

In this experiment, you will study the behavior of the damped oscillator, and explore two different ways of determining the quality factor Q. You will also explore the effects of different types of damping.

2 THEORY

2.1 Undamped Oscillations

First, consider the case of undamped oscillations. The restoring torque τ is proportional to the angular displacement $\theta = (\theta_{raw} - \theta_R)$:

$$\tau = -\kappa\theta\,.$$

Using Newton's second law in rotational form,

$$\tau = I\alpha = I\frac{d^2\theta}{dt^2} = I\ddot{\theta}\,,$$

using the notation that $\dot{\theta} = \frac{d\theta}{dt}$, and $\ddot{\theta} = \frac{d^2\theta}{dt^2}$. Combining these two gives

$$I\ddot{\theta} = -\kappa\theta\,. \tag{1}$$

Dividing through by I gives the canonical form for a harmonic oscillator

$$\ddot{\theta} = -\omega_0^2 \theta \,. \tag{2}$$

where $\omega_0 = \frac{\kappa}{I}$.

The solution to this differential equation is simple harmonic motion,

$$\theta = \theta_0 \cos(\omega_0 t + \phi) \tag{3}$$

where θ_0 is the amplitude of the motion and ϕ is a phase constant; these constants are set by the initial position and velocity. The constant ω_0 is called the *undamped* angular frequency, and it is related to the linear frequency f and the period T by

$$\omega_0 = 2\pi f = \frac{2\pi}{T} \,.$$

2.2 Linear Damping

For many common cases, an oscillator is subject to a damping force or torque that depends on velocity. Common examples are viscous drag and magnetic damping.

For the torsional oscillator, this means adding a drag torque proportional to $\dot{\theta}$:

$$\ddot{\theta} = -\omega_0^2 \theta - \gamma \dot{\theta} \,, \tag{4}$$

where γ is a term proportional to the drag, and has units of 1/s. The minus sign indicates that the drag opposes the motion.

For reasonably small damping (specifically, $\gamma < 2\omega_0$), the solution to Eq. 4 is a *damped* harmonic oscillator,

$$\theta(t) = \theta_0 e^{-\gamma t/2} \cos(\omega_v t + \phi) \tag{5}$$

where γ is the damping, and ω_v is the angular frequency of the *damped* oscillator. This can be thought of as an oscillation, but with a damped amplitude $\theta_0 e^{-\gamma t/2}$.

The *damped* frequency ω_v is related to the *undamped* natural angular frequency ω_0, by

$$\omega_v = \omega_0 \sqrt{1 - \frac{\gamma^2}{4\omega_0^2}} \,. \tag{6}$$

although for cases of small damping, the difference between ω_0 and ω_v is often too small to be reliably measured.

It will also be convenient to define a *quality factor* Q by

$$Q = \frac{\omega_0}{\gamma} \,. \tag{7}$$

A large value of Q corresponds to very weak damping, while a smaller value of Q corresponds to larger damping.

2.3 Air Drag

One other type of damping commonly encountered is air drag. Instead of being proportional to velocity, the damping is proportional to velocity *squared*. Thus the equation of motion becomes

$$\ddot{\theta} = -\omega_0^2\theta - c|\dot{\theta}|\dot{\theta}\,, \tag{8}$$

where c is proportional to the drag coefficient and the combination $-|\dot{\theta}|\dot{\theta}$ ensures that the drag has a magnitude proportional to the velocity squared, but acts in a direction to oppose the motion.

For the case of air drag, the oscillator will oscillate with a decaying amplitude, but there is no analytic solution.

3 DATA ACQUISITION

3.1 Initial Setup

Use four of the brass quadrants to give the system a reasonably large moment of inertia. (You will have to stack them two high to leave the holes free for the air paddles in §3.4 below.) Use `LoggerPro` to measure the output voltage corresponding to the angular displacement. Let the oscillator come to equilibrium, and adjust the "Zero Adjust" so that the output voltage is close to zero.

Use the following initial settings on `LoggerPro`: Under `Experiment` → `Data Collection`, set the `Experiment Length` to 30 seconds, and the `Sampling Rate` to 500 samples/second. Feel free to adjust these values during the course of the experiment, but these are good for a start.

Connect the DC power supply to the terminals marked "Coil Drive." Throughout this experiment, *keep the total current under 2A!* You will use the coil drive to set an initial angle and release the rotor from rest. For now, set the drive current to zero.

3.2 Undamped Oscillations

The first task is to measure ω_0, the frequency of *undamped* oscillations. Back out the magnetic brakes so they are as far away from the copper rotor as possible. In order to completely remove the possibility of magnetic damping from the system, you may even remove both magnet assemblies from the apparatus, provided you do so gently, and are very careful to align them well when you put them back. This makes a very small, but measurable, difference.

Gradually turn on the current to about 0.50 A and let the rotor settle down. Make sure it is not wobbling. Roughly what is the initial angular displacement? Is that within the range of linear voltage response you found in the previous experiment? If not, make adjustments as appropriate.

Toggle the `Output` On/Off switch to turn off the drive current so that the rotor starts oscillating. Use `LoggerPro` to record the position as a function of time for 30 seconds. There is a time delay between when you press the `Start` button and when `LoggerPro` starts collecting data. One way to avoid this is to use *triggering*. (This is available under the `Experiment` → `Data Collection` menu. Select the tab for `Triggering`.) Turn the DC current back on to about 0.50 A and let the rotor settle again. Note the steady value of the `LoggerPro` voltage before you turn off the current, and set the trigger value to a value closer to zero. Pick the appropriate direction (increasing or decreasing). Press the green `Start` button. `LoggerPro` will wait until the triggering condition is satisfied. Now turn off the power supply to start the oscillation.

When you are satisfied, save your work. (This will save all the `LoggerPro` settings as well as the data.) Also use `File` → `Export As` → `CSV` to export the data to a file easily imported into other applications such as *Mathematica*.

To test for reproducibility, repeat the data collection 4 times so you have a total of 5 data files stored.

3.3 Damped Oscillations

Now move the magnetic brakes closer to the rotor. The exact position is not critical. Adjusting the magnets until the leading edge is just over the outer edge of the copper rotor seems to work fine. Be sure the magnets are not touching the rotor.

You have now introduced significant damping into the system. Gradually turn on the current to about 1.50 A and let the rotor settle down.[1] Make sure it is not wobbling. Adjust the triggering to reflect the new initial voltage level. Release the rotor from rest, and record the angular position as a function of time. If there is heavy damping, you may stop the collection earlier than 30 s, or just use the first part of the data, or you may adjust the position of the magnetic brakes and try again.

Again, save your data and repeat so you have a total of 5 damped trials.

3.4 Oscillations with Air Drag

Back the magnets all the way out again. (It is not useful to remove them for this part.) Insert the two air paddles into the holes above the rotor. (See the instruction manual for

[1] The initial displacement is chosen so that the *average* angular displacement in this damped case will be about the same as the average angular displacement in the previous undamped case. This will tend to minimize any amplitude-dependent effects on the period.

photos.) The exact placement is not critical, but placing the paddles about 3/4 of the way out seems to work well.

Set the coil voltage to the largest initial angle you can use before the paddles hit the wooden frame. Adjust the triggering voltage appropriately. Release the rotor from rest and record the voltage as a function of time. Save your data. A single trial is sufficient here.

4 ANALYSIS

4.1 Undamped Oscillations

4.1.1 Conversion to Angle

Refer to your previous work where you found a calibration function to convert voltage to angle. (The intercept may have shifted, but we will account for that below.) Import your data from one of your undamped trials and convert the voltage readings into angular displacements.

For consistency with later parts of the experiment, fit your data to a function of the form:

$$\theta(t) = \theta_0 e^{-\gamma t/2} \cos(\omega t + \phi) + \theta_{\text{off}} \tag{9}$$

where θ_{off} accounts for any residual zero offset.

Ideally, the damping should be 0, so the angular frequency you measure this way is ω_0. Record ω_0 and the uncertainty in that value. Record the actual value of γ (and its uncertainty) as well. *Once you have measured damped oscillations, come back and evaluate whether this amount of damping was really "small."*

Note: You will probably find a tiny uncertainty in ω_0, often on the order of 1×10^{-5} rad/s. The actual frequency is not always quite that reproducible. To check, repeat that fit for all 5 of your undamped data runs so you can compute an average ω_0 and its uncertainty.

Include a graph showing one of your undamped runs, along with a fit to the data.

4.2 Damped Oscillations

Import one of your runs for a damped oscillation, convert to angles, and fit Eq. 9 to the damped data. Report the values for ω_v and γ, along with their uncertainties. Include a graph showing one of your damped runs, along with a fit to the data.

As before, repeat the fit for your 5 trials and find the average ω_v, along with its uncertainty.

4.3 Determining Q

4.3.1 Comparing damped and undamped frequencies.

Substitute Eq. 7 into Eq. 6 to eliminate γ, and solve for Q. Find Q. Note that both ω_0 and ω_v have uncertainty. Is your precision in them high enough to warrant doing this calculation? (To be specific, compare the difference between ω_0 and ω_v to the uncertainties in each.) If so, give your result for Q and its uncertainty. If not, then explain why you can't do this calculation.

4.3.2 Using the damping coefficient γ

Using your fit for the damping coefficient γ, evaluate Q directly from Eq. 7, and give your result for both Q and its uncertainty.

4.4 Air Drag

Finally, import your data for air drag and attempt to fit Eq. 9. Is Eq. 9 a good model, or is the behavior significantly different?

4.5 Overall

In this lab, you calculated Q in two different ways: from the frequency of damped oscillations, and from the decay of free vibrations. Compare your two values.