AC Filters part II: Fourier Perspective

In[417]:=
```
Clear["Global`*"]; DateString[]
SetDirectory[NotebookDirectory[]];
```
Out[417]=
```
Tue 29 Apr 2025 20:24:37
```

FFT Measurements on Input 5 kHz Waveforms

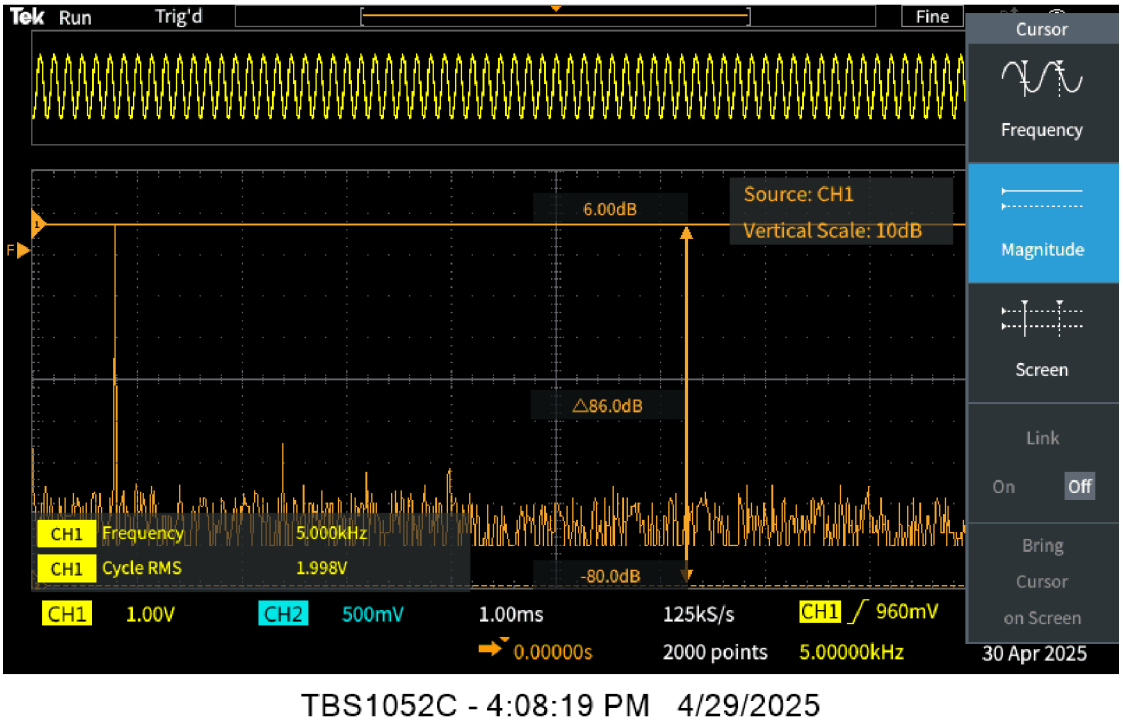
Convert between Voltage and dB scales. dB = 20 Log10(V/Vref), where Vref is 1 V. The oscilloscope uses RMS voltages for this calculation, not peak-to-peak.

In[419]:=
```
dBtoV[data_] := Map[{#[[1]], 10^(#[[2]] / 20)} &, data]
```
In[420]:=
```
VtodB[data_] := Map[{#[[1]], 20. Log10[#[[2]]]} &, data]
```

Sine Waves

Input signal is 2 Vrms @ 5000 Hz.

In[421]:=
```
Import["Sine-dB-FFT.png"]
```
Out[421]=



In[422]:=
```
Vrms = 2.0;
dB = 6.0;
20 Log10[Vrms]
```
Out[424]=
```
6.0206
```

Triangle Waves

Triangle waves, 4Vpp, 5000 Hz: {mode, frequency, dBin}

In[425]:=
```
triangle = {{1, 5000, 1.80}, {3, 15000, -18.4}, {5, 25000, -27.2}, {7, 35000, -32.8}, {9, 45000, -37.6}};
```

In[426]:=
```
triamps = dBtoV[triangle[[All, {1, 3}]]];
```

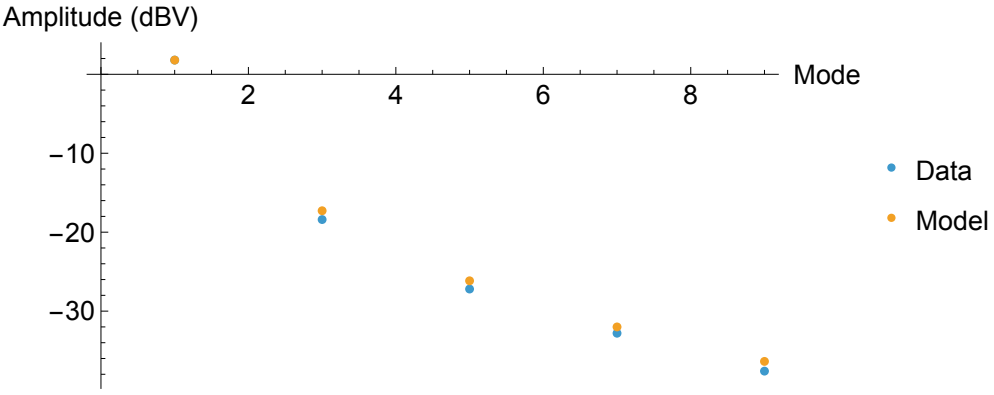
In[427]:=
```
A0 = triamps[[1, 2]];
```

In[428]:=
```
trimodel = Table[{n, A0 / n^2}, {n, {1, 3, 5, 7, 9}}];
```

In[429]:=
```
tridBmodel = VtodB[trimodel];
```

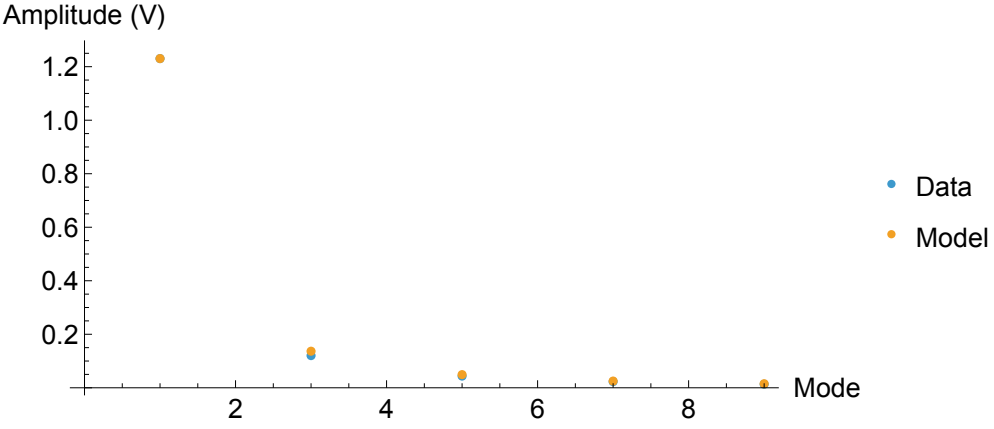
In[430]:=
```
(* dB Plot: *)
ListPlot[{triangle[[All, {1, 3}]], tridBmodel}, PlotLegends → {"Data", "Model"},
 LabelStyle → Larger, AxesLabel → {"Mode", "Amplitude (dBV)"},
 AspectRatio → 1 / 2, ImageSize → Scaled[0.7]]
```

Out[430]=



In[431]:=
```
(* Linear scale plot: *)
ListPlot[{triamps, trimodel}, PlotLegends → {"Data", "Model"},
 LabelStyle → Larger, AxesLabel → {"Mode", "Amplitude (V)"},
 AspectRatio → 1 / 2, ImageSize → Scaled[0.7]]
```

Out[431]=



Square Waves

Square waves, 4.4Vpp, 5000 Hz: {mode, frequency, dBin}

In[432]:=

$$\text{square} = \begin{pmatrix} 1 & 5000 & 0. \\ 3 & 15\,000 & -10.8 \\ 5 & 25\,000 & -15.2 \\ 7 & 35\,000 & -18.4 \\ 9 & 45\,000 & -20.0 \end{pmatrix};$$

In[433]:=

```
sqamps = dBtoV[square[[All, {1, 3}]]];
```

In[434]:=

```
A0 = sqamps[[1, 2]];
```

In[435]:=

```
sqmodel = Table[{n, A0 / n}, {n, {1, 3, 5, 7, 9}}];
```

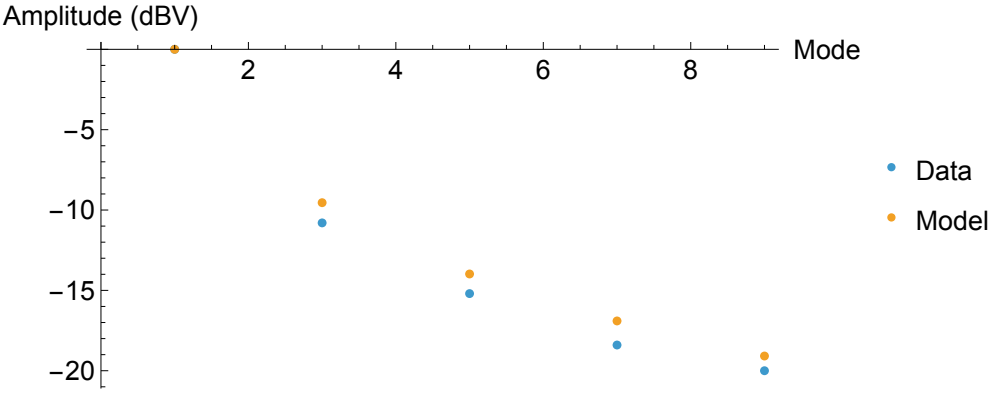
In[436]:=

```
sqdBmodel = VtodB[sqmodel];
```

In[437]:=

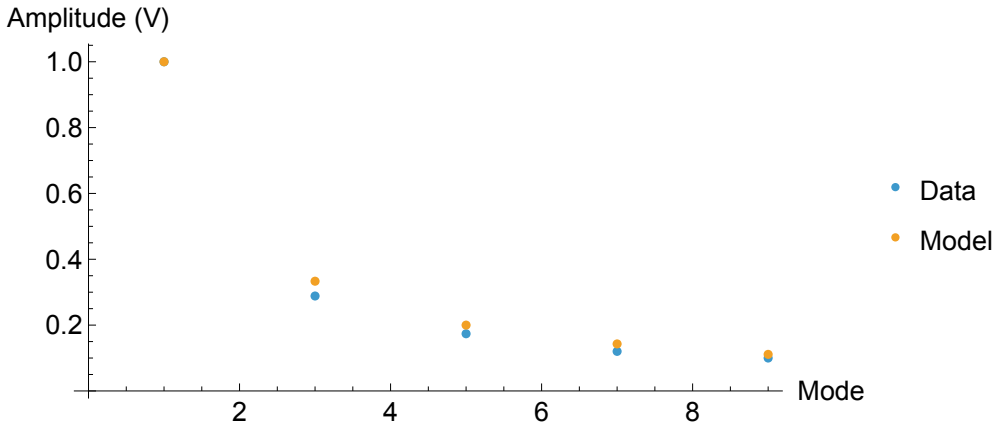
```
(* dB Plot: *)
ListPlot[{square[[All, {1, 3}]], sqdBmodel}, PlotLegends → {"Data", "Model"},
 LabelStyle → Larger, AxesLabel → {"Mode", "Amplitude (dBV)"},
 AspectRatio → 1 / 2, ImageSize → Scaled[0.7]]
```

Out[437]=



In[438]:=
```
(* Linear scale plot: *)
ListPlot[{sqamps, sqmodel}, PlotLegends → {"Data", "Model"},
 LabelStyle → Larger, AxesLabel → {"Mode", "Amplitude (V)"},
 AspectRatio → 1 / 2, ImageSize → Scaled[0.7]]
```

Out[438]=



Low-Pass Filter: FFT Measurements

Component Values

In[439]:=
```
rR = 3255 (* Resistor *); rFG = 600 (* Function Generator *); r = rR;
```

In[440]:=
```
c = 22.4 * 10^(-9); (* Capacitor *)
```

In[441]:=
```
fbComponents = 1 / (2 * π * r * c)
```

Out[441]=
```
2182.84
```

In[442]:=
```
ALowPass[fb_, f_] := 1 / Sqrt[1 + (f / fb)^2]
 (* Theoretical gain expressed in terms of breakpoint f *)
```

Triangle Waves

Low Frequency Triangle Waves: f = 220 Hz

Triangle waves, 4Vpp, 220 Hz: {mode, dBin, dBout}

In[443]:=
```
f1 = 220. (* Fundamental *);
```

In[444]:=

$$\text{triangle} = \begin{pmatrix} 1 & 1.20 & 1.20 \\ 3 & -18.0 & -18.4 \\ 5 & -26.9 & -28.4 \\ 7 & -32.8 & -34.4 \\ 9 & -38.0 & -40 \end{pmatrix};$$

In[445]:=

```
tridBin = Map[{f1 * #[[1]], #[[2]]} &, triangle];
triVin = dBtoV[tridBin];
```

In[447]:=

```
tridBout = Map[{f1 * #[[1]], #[[3]]} &, triangle];
triVout = dBtoV[tridBout];
```

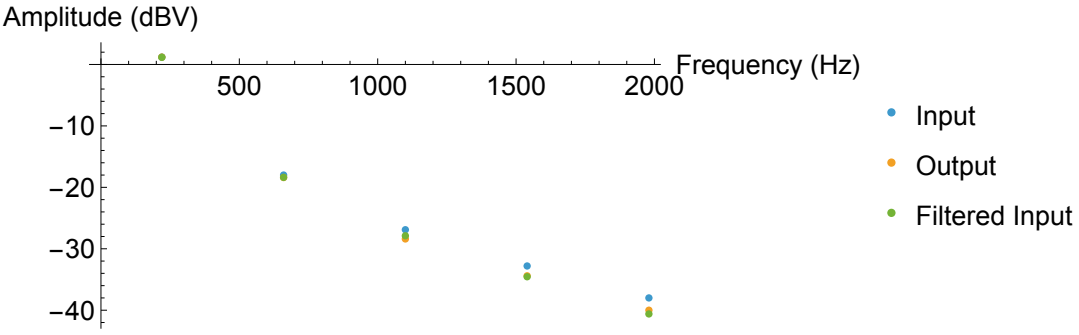
In[449]:=

```
(* Pass each input mode through the low pass filter *)
triVmodel = Map[{#[[1]], #[[2]] * ALowPass[fbComponents, #[[1]]]} &, triVin];
tridBmodel = VtodB[triVmodel];
```

In[451]:=

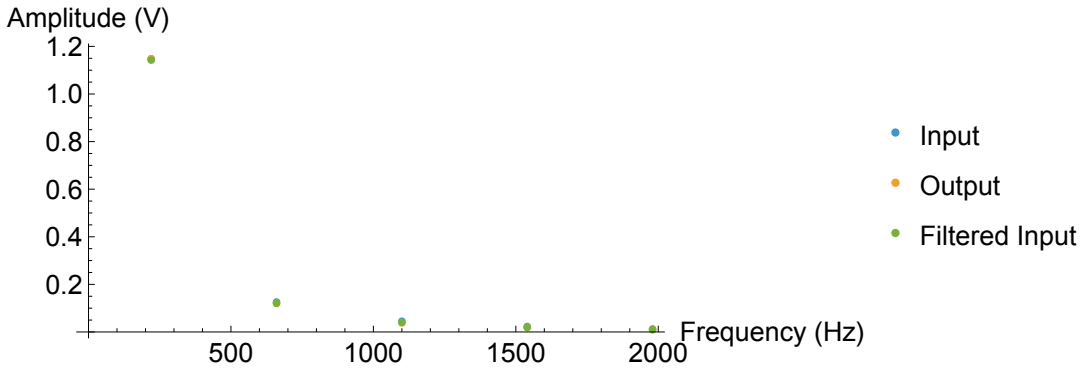
```
ListPlot[{ tridBin, tridBout, tridBmodel},
 PlotLegends → {"Input", "Output", "Filtered Input" },
 LabelStyle → Larger, AxesLabel → {"Frequency (Hz)", "Amplitude (dBV)"},
 AspectRatio → 1 / 2, ImageSize → Scaled[0.7]]
```

Out[451]=



In[452]:=
```
ListPlot[{triVin, triVout, triVmodel},
 PlotLegends → {"Input", "Output", "Filtered Input"},
 LabelStyle → Larger, AxesLabel → {"Frequency (Hz)", "Amplitude (V)"},
 AspectRatio → 1 / 2, ImageSize → Scaled[0.7]]
```

Out[452]=



Medium Frequency Triangle Waves: f = 2200 Hz

Triangle waves, 4Vpp, 2200 Hz: {mode, dBin, dBout}

In[453]:=
```
f1 = 2200. (* Fundamental frequency *);
```

In[454]:=
```
triangle = {{1, 1.20, -2.0}, {3, -18.4, -28.4}, {5, -27.6, -39.6}, {7, -33.6, -51.2}, {9, -37.6, -53.2}};
```

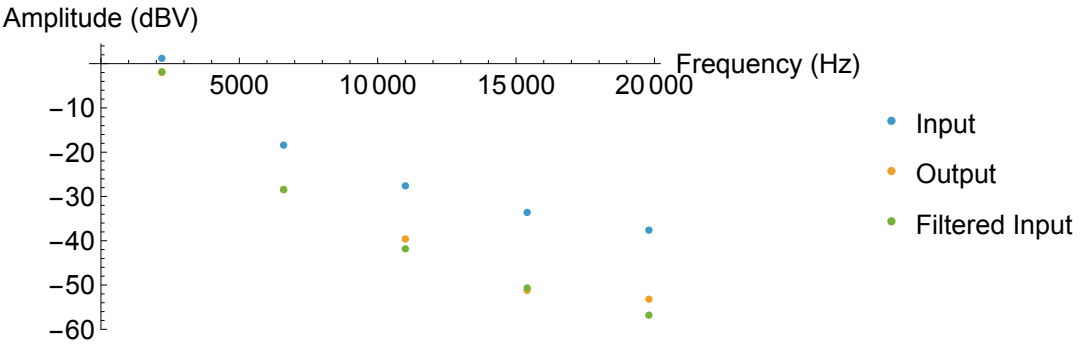
In[455]:=
```
tridBin = Map[{f1 * #[[1]], #[[2]]} &, triangle];
triVin = dBtoV[tridBin];
```

In[457]:=
```
tridBout = Map[{f1 * #[[1]], #[[3]]} &, triangle];
triVout = dBtoV[tridBout];
```

In[459]:=
```
(* Pass each input mode through the low pass filter *)
triVmodel = Map[{#[[1]], #[[2]] * ALowPass[fbComponents, #[[1]]]} &, triVin];
tridBmodel = VtodB[triVmodel];
```

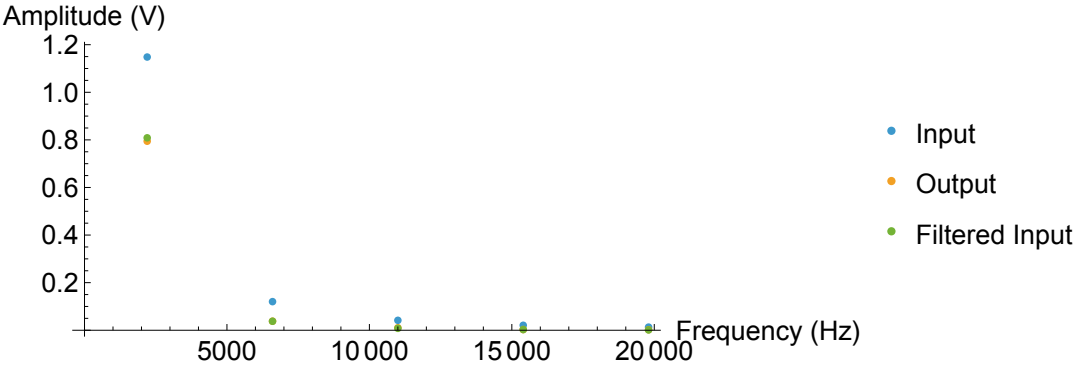
In[461]:=
```
ListPlot[{ tridBin, tridBout, tridBmodel},
 PlotLegends → {"Input", "Output", "Filtered Input" },
 LabelStyle → Larger, AxesLabel → {"Frequency (Hz)", "Amplitude (dBV)"},
 AspectRatio → 1 / 2, ImageSize → Scaled[0.7]]
```

Out[461]=



In[462]:=
```
ListPlot[{triVin, triVout, triVmodel},
 PlotLegends → {"Input", "Output", "Filtered Input"},
 LabelStyle → Larger, AxesLabel → {"Frequency (Hz)", "Amplitude (V)"},
 AspectRatio → 1 / 2, ImageSize → Scaled[0.7]]
```

Out[462]=



High Frequency Triangle Waves: f = 22000 Hz

Triangle waves, 4Vpp, 22,000 Hz: {mode, dBin, dBout}

In[463]:=
```
f1 = 22 000. (* Fundamental frequency *);
```

In[464]:=
```
triangle = {{1, 1.20, -18.8}, {3, -18.0, -49.6}, {5, -27.6, -57.2}, {7, -34.0, -65.}, {9, -38.4, -68.0}};
```

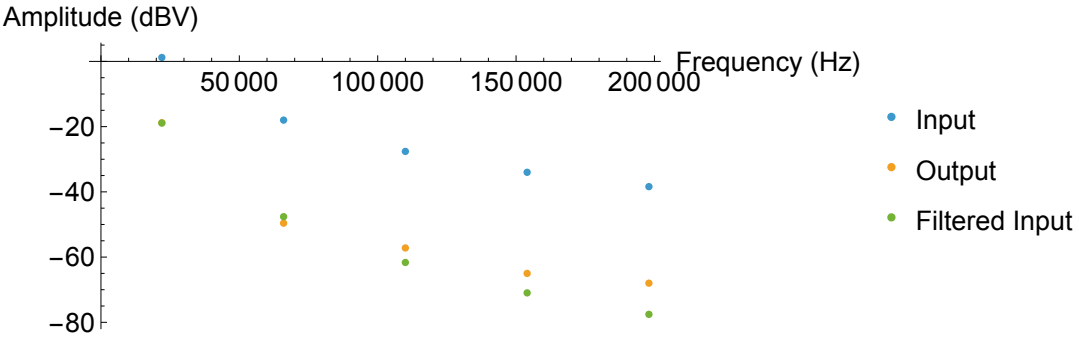
In[465]:=
```
tridBin = Map[{f1 * #[[1]], #[[2]]} &, triangle];
triVin = dBtoV[tridBin];
```

In[467]:=
```
tridBout = Map[{f1 * #[[1]], #[[3]]} &, triangle];
triVout = dBtoV[tridBout];
```

In[469]:=
```
(* Pass each input mode through the low pass filter *)
triVmodel = Map[{#[[1]], #[[2]] * ALowPass[fbComponents, #[[1]]]} &, triVin];
tridBmodel = VtodB[triVmodel];
```

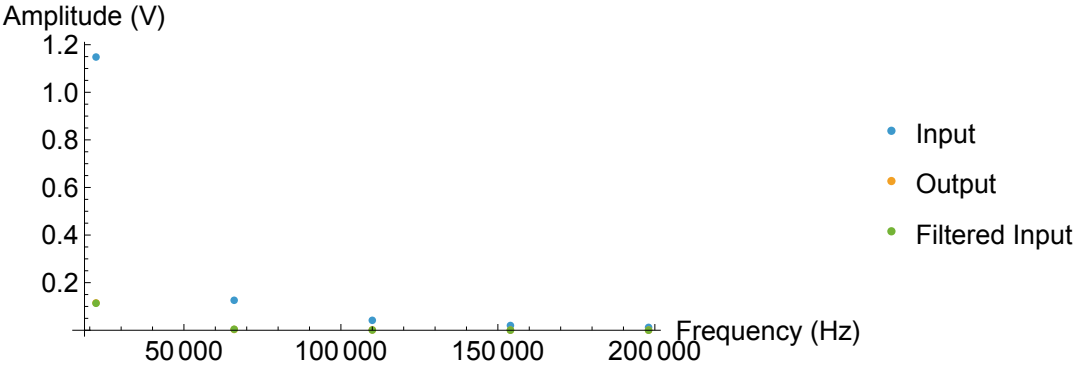
In[471]:=
```
ListPlot[{ tridBin, tridBout, tridBmodel},
 PlotLegends → {"Input", "Output", "Filtered Input" },
 LabelStyle → Larger, AxesLabel → {"Frequency (Hz)", "Amplitude (dBV)"},
 AspectRatio → 1 / 2, ImageSize → Scaled[0.7]]
```

Out[471]=



In[472]:=
```
ListPlot[{triVin, triVout, triVmodel},
 PlotLegends → {"Input", "Output", "Filtered Input"},
 LabelStyle → Larger, AxesLabel → {"Frequency (Hz)", "Amplitude (V)"},
 AspectRatio → 1 / 2, ImageSize → Scaled[0.7], PlotRange → All]
```

Out[472]=



Square Waves

Low Frequency Square Waves: f = 220 Hz

Square waves, 2Vpp, 220 Hz: {mode, dBin, dBout}

In[473]:=
```
f1 = 220. (* Fundamental frequency *);
```

In[474]:=
```
square = {{1, -1.20, -1.20}, {3, -10.8, -11.2}, {5, -15.6, -16.0}, {7, -18.4, -20}, {9, -20.8, -23.2}};
```

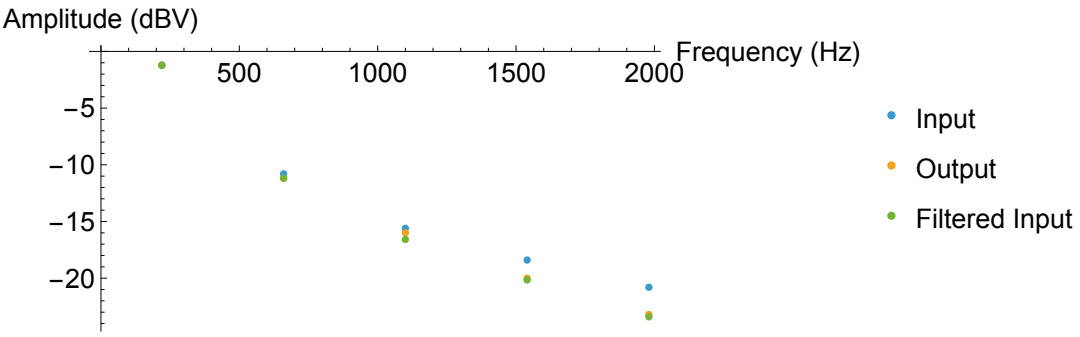
In[475]:=
```
sqdBin = Map[{f1 * #[[1]], #[[2]]} &, square];
sqVin = dBtoV[sqdBin];
```

In[477]:=
```
sqdBout = Map[{f1 * #[[1]], #[[3]]} &, square];
sqVout = dBtoV[sqdBout];
```

In[479]:=
```
(* Pass each input mode through the low pass filter *)
sqVmodel = Map[{#[[1]], #[[2]] * ALowPass[fbComponents, #[[1]]]} &, sqVin];
sqdBmodel = VtodB[sqVmodel];
```

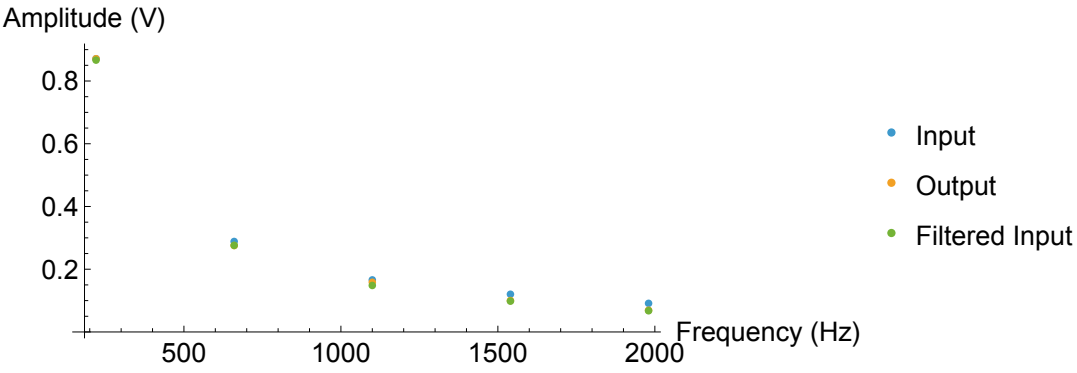
In[481]:=
```
ListPlot[{ sqdBin, sqdBout, sqdBmodel},
 PlotLegends → {"Input", "Output", "Filtered Input" },
 LabelStyle → Larger, AxesLabel → {"Frequency (Hz)", "Amplitude (dBV)"},
 AspectRatio → 1 / 2, ImageSize → Scaled[0.7]]
```

Out[481]=



In[482]:=
```
ListPlot[{sqVin, sqVout, sqVmodel},
 PlotLegends → {"Input", "Output", "Filtered Input"},
 LabelStyle → Larger, AxesLabel → {"Frequency (Hz)", "Amplitude (V)"},
 AspectRatio → 1 / 2, ImageSize → Scaled[0.7], PlotRange → All]
```

Out[482]=



Medium Frequency Square Waves: f = 2200 Hz

square waves, 2Vpp, 2200 Hz: {mode, dBin, dBout}

In[483]:=
```
f1 = 2200. (* Fundamental frequency *);
```

In[484]:=
```
square = {{1, -1.60, -4.40}, {3, -12.0, -21.6}, {5, -16.4, -30}, {7, -19.6, -36}, {9, -21.2, -41.2}};
```

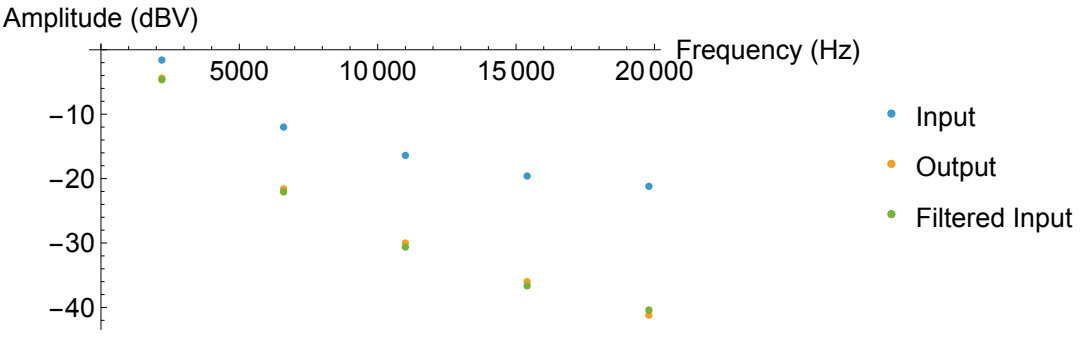
In[485]:=
```
sqdBin = Map[{f1 * #[[1]], #[[2]]} &, square];
sqVin = dBtoV[sqdBin];
```

In[487]:=
```
sqdBout = Map[{f1 * #[[1]], #[[3]]} &, square];
sqVout = dBtoV[sqdBout];
```

In[489]:=
```
(* Pass each input mode through the low pass filter *)
sqVmodel = Map[{#[[1]], #[[2]] * ALowPass[fbComponents, #[[1]]]} &, sqVin];
sqdBmodel = VtodB[sqVmodel];
```

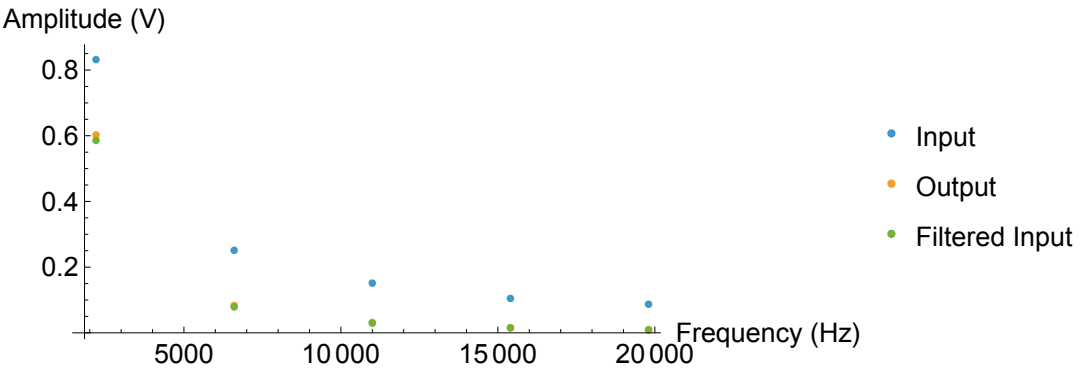
In[491]:=
```
ListPlot[{ sqdBin, sqdBout, sqdBmodel},
 PlotLegends → {"Input", "Output", "Filtered Input" },
 LabelStyle → Larger, AxesLabel → {"Frequency (Hz)", "Amplitude (dBV)"},
 AspectRatio → 1 / 2, ImageSize → Scaled[0.7]]
```

Out[491]=



In[492]:=
```
ListPlot[{sqVin, sqVout, sqVmodel},
 PlotLegends → {"Input", "Output", "Filtered Input"},
 LabelStyle → Larger, AxesLabel → {"Frequency (Hz)", "Amplitude (V)"},
 AspectRatio → 1 / 2, ImageSize → Scaled[0.7], PlotRange → All]
```

Out[492]=



High Frequency Square Waves: f = 22000 Hz

square waves, 2Vpp, 22,000 Hz: {mode, dBin, dBout}

In[493]:=
```
f1 = 22 000. (* Fundamental frequency *);
```

In[494]:=
```
square = {{1, -1.60, -21.2}, {3, -11.2, -40.4}, {5, -16, -50}, {7, -19.2, -57.2}, {9, -21.6, -62.4}};
```

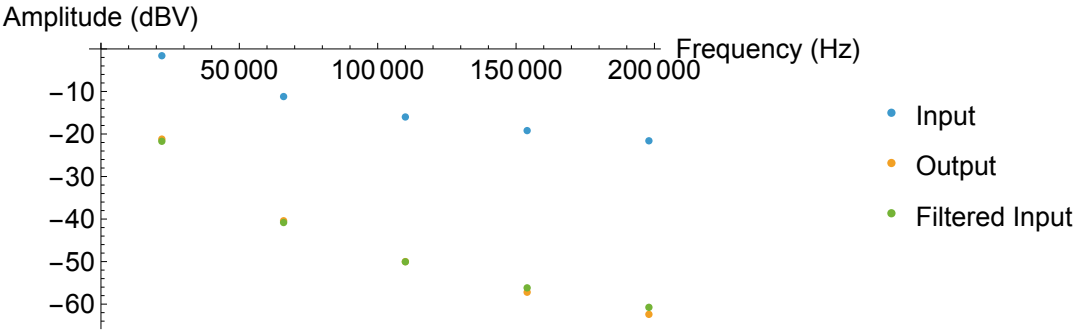
In[495]:=
```
sqdBin = Map[{f1 * #[[1]], #[[2]]} &, square];
sqVin = dBtoV[sqdBin];
```

In[497]:=
```
sqdBout = Map[{f1 * #[[1]], #[[3]]} &, square];
sqVout = dBtoV[sqdBout];
```

In[499]:=
```
(* Pass each input mode through the low pass filter *)
sqVmodel = Map[{#[[1]], #[[2]] * ALowPass[fbComponents, #[[1]]]} &, sqVin];
sqdBmodel = VtodB[sqVmodel];
```

In[501]:=
```
ListPlot[{ sqdBin, sqdBout, sqdBmodel},
 PlotLegends → {"Input", "Output", "Filtered Input" },
 LabelStyle → Larger, AxesLabel → {"Frequency (Hz)", "Amplitude (dBV)"},
 AspectRatio → 1 / 2, ImageSize → Scaled[0.7]]
```

Out[501]=



In[502]:=
```
ListPlot[{sqVin, sqVout, sqVmodel},
 PlotLegends → {"Input", "Output", "Filtered Input"},
 LabelStyle → Larger, AxesLabel → {"Frequency (Hz)", "Amplitude (V)"},
 AspectRatio → 1 / 2, ImageSize → Scaled[0.7], PlotRange → All]
```

Out[502]=