Physics 327—Advanced Classical Mechanics
Problem Set #1
Due Wednesday, January 31, 2024, 2:45 p.m.

All problems are due at the *beginning* of class on Wednesday.

Text Problems

Taylor: 2.7, 2.8, 2.12, 2.28, and 2.36. These problems will be worth 20 points each. Also do the two supplementary problems below.

Hint: Problem 2.36 is probably most easily done in *Mathematica*. You may find it helpful to do the two supplementary problems below before trying problem 2.36.

Supplementary Problems

For a number of problems in this course, we will find it convenient to use *Mathematica*. These two problems are designed both to address specific issues of motion subject to a quadratic drag force, and to help you become more familiar with using *Mathematica* in physics problems. Accordingly, I have provided you with templates on our Moodle page that are very verbose. Please work through them carefully and consult the on-line help often if you have questions. I will also be happy to help you out. You should find that the actual calculations required for these problems are almost completely done for you already. You only have to make minor adjustments to the notebooks I have provided. Still, I hope you become comfortable doing such calculations; future assignments will leave a lot more of the details to you.

Problem S1 – Vertical Motion With Quadratic Drag

(40 pts.) Consider the vertical motion of a dropped baseball. Use the parameters from Example 2.5 in the text. The motion of the ball is governed by the differential equation

$$\ddot{y} = g\left(1 - \frac{\dot{y}^2}{v_{term}^2}\right) \tag{1}$$

The difficulty with this equation is that the velocity $\dot{y}$ appears on the right-hand side, but isn't necessarily known yet. In a few such problems, there is an analytical solution. In many cases, however, you need to make some sort of approximation. As we will discuss in class, one approach to solving this problem *approximately* is to replace the correct velocity on the right-hand side with the velocity you would get if there were no air drag. If the overall effect of air drag is small, this will be a reasonable approximation. Your assignment in this problem is to compare that approximation with the exact solution.

Specifically, consider a baseball dropped from an initial height $h = 50.0\,\text{m}$, and consider two solutions: the exact solution, and an approximate solution where $\dot{y}$ on the right hand side of Eq. 1 is replaced by the velocity you would have if there were no air drag.

a. Errors in time. (20 pts.) Find the time it takes to reach the ground for both the exact and approximate solutions. What is the percent error in the time for the approximate solution? Is that likely to be noticeable?

b. Errors in velocity. (20 pts.) Find the velocity of the ball when it reaches the ground for both the exact and approximate solutions. What is the percent error in the velocty for the approximate solution? Is that likely to be noticeable?

Problem S2 – Projectile Motion With Quadratic Drag

(40 pts.) Consider a baseball thrown at an angle θ with an initial speed of v_0. Assuming that the ball is subject to the quadratic drag force of Eq. 2.60, what is the angle θ that maximizes the range of the baseball for each of the following two conditions? Give your answer to a precision of $\pm 0.1°$.

a. (20 pts.) For an initial speed of $30\,\text{m/s}$, what is the angle θ that maximizes the range of the baseball? Give your answer to a precision of $\pm 0.1°$.

b. (20 pts.) For an initial speed of $15\,\text{m/s}$, what is the angle θ that maximizes the range of the baseball? Give your answer to a precision of $\pm 0.1°$. Comment briefly on the qualitative trend you observe between these two velocities.

Academic Honesty

You may use, without proof, any results from class or from your text by simply quoting the result and giving the reference (*e.g.* equation number or page number). You should understand *how* that result was obtained, but you need not transcribe the derivation.

If you get bogged down with any of the problems, do not hesitate to discuss them with me or with a fellow student. However, if you discuss a problem with *anyone* (besides me) you should acknowledge that collaboration.