Physics 327—Advanced Classical Mechanics
Problem Set #2
Due Wednesday, February 7, 2024, 2:45 p.m.

All problems are due at the *beginning* of class on Wednesday.

Text Problems

Ch. 6: 6.2, 6.5, 6.7, and 6.22.

Ch. 7: 7.2 and 7.3

One-star problems count 10 points each; two-star problems count for 20 points, and three-star problems count for 30 points.

Hints

6.2. Note that you don't have to evaluate any integrals here – you are just setting up an integral. For comparison, note that the answer to problem 6.1 is given in Eq. 6.41.

6.5. This is a straight calculus problem, not a calculus of variations problem. The path is assumed to consist of two straight-line segments. The only variable is the angle θ.

6.7. This problem builds on your answer to problem 6.2. The question "Is it unique?" is neither well-phrased nor particularly deep. You may safely skip the uniqueness question. There are two main issues involved: First, when specifying a point on the cylinder by its angle ϕ, one can always add or subtract multiples of 2π without changing the physical location. However, in terms of your geodesic, you might consider such multiples as constraining you to wind around the cylinder multiple times on the way from the start point to the end point. Second, when going from ϕ_1 to ϕ_2, you have a choice of going clockwise or counter-clockwise. Both choices would be local minima of the length integral.

6.22. The area is given, as usual, by $A = \int y dx$, but the constraint in this problem is on the total length of the string, $L = \int ds$. It is probably best to think of s as the independent variable. That is, think of the path as $y(s)$, and let $y'(s) = \frac{dy}{ds}$. You will then have to replace dx by an expression in terms of $y'(s)$ and ds. (This is just the inverse of the usual substitution we did for ds in terms of dx.

You may use the hint in the problem or you may skip it. In any case, you will end up with a differential equation. It's probably easiest to solve by guessing the answer: $y(s) = R\sin(s/R)$. You may plug that in and verify that it satisfies the differential equation. (At the moment, R is just some unknown constant. It will turn out to be the radius of the circle.)

Next, you need to convert $y(s)$ into more familiar form $y(x)$. That's most readily accomplished by the intermediate step of finding $x(s)$. Recall above you got dx in terms of ds and $y'(s)$. Just integrate that expression to get $x(s)$. You now have a parametric expression for the path $x(s), y(s)$. Again, use your knowledge of the answer to suggest what to do. What is the equation for a circle centered at $x = R$, $y = 0$? Do your expressions for $x(s)$ and $y(s)$ satisfy that equation? If so, then you are done!

7.2 and **7.3**. These are straight-forward exercises intended to make you more familiar with the Lagrangian approach.

Academic Honesty

You may use, without proof, any results from class or from your text by simply quoting the result and giving the reference (*e.g.* equation number or page number). You should understand *how* that result was obtained, but you need not transcribe the derivation.

If you get bogged down with any of the problems, do not hesitate to discuss them with me or with a fellow student. However, if you discuss a problem with *anyone* (besides me) you should acknowledge that collaboration.