Physics 327—Advanced Classical Mechanics
Problem Set #6
Due Wednesday, March 27, 2024, 2:45 p.m.

All problems are due at the *beginning* of class on Wednesday.

Text Problems

Ch. 9: 9.29.

Ch. 10: 10.5, 10.6, 10.11, 10.22, and 10.26

Supplemental Problem: Numerical Simulations of Gravitational Orbits

(Originally from problem set 5.)

For these problems, you should start with the *Mathematica* notebook `https://workbench.lafayette.edu/~doughera/courses/phys327-2024/homework/ps05/Ch08-orbits-3body-template.nb` available from the course web site.

The notebook simulates the interactions of a moon with the sun and a planet orbiting around the sun in a circular orbit. No extensive programming is required, but you will have to read and make minor modifications to the existing notebook.

What to submit: You do not need to submit a *Mathematica* notebook. The questions below call for simple measurements or qualitative descriptions of your observations from the simulations. Simply report those measurements or observations. Your submission should be quite short.

a. (Warm-up.) All calculations will be done in "Solar Units." Read through the first part of the notebook. The mass of the sun is set to 1.0. Set $M_{\text{planet}} = M_{\text{sun}}/500$. Set the radius of the planet's orbit to 1.0. The first part only includes the sun and planet. Run the simulation. What is the approximate period T_{planet} of the planet's orbit? It should be approximately 1 year in these units.

b. (10 pts.) Next, we will add a moon. The moon will orbit the sun, but at least initially, we will ignore interactions of the moon with the planet. Place the moon at an initial angle of 180° at an initial radius such that it orbits the sun with a period of $\frac{2}{3}T_{planet}$. What initial radius do you need? (The notebook will automatically compute the needed velocity.) Run the simulation and verify the orbital period.

c. (10 pts.) Now add in the interaction with the planet. (See the instructions in the *Mathematica* notebook.) Continue to assume that $M_{\text{planet}} = M_{\text{sun}}/500$. Run the simulation for $tmax = 10T_{\text{planet}}$. What is the new approximate period of the moon's orbit? Is the orbit still approximately circular?

d. (10 pts.) Change the initial radius of the moon's orbit to 0.85, and run the simulation for at least 5 years. (You may have to slow the simulation down or use the single-step buttons.) Describe qualitatively what happens to the moon now. Is the orbit still approximately circular?

e. (10 pts.) Place the moon at the L2 Lagrange point. (The notebook changes the `alleqn` function to set the initial conditions correctly.) Run the simulation. Does the Lagrange point appear to be stable? You are not looking for a proof here, just for qualitative observations to support your conclusion. (*Hint:* Try changing the initial conditions very slightly (e.g. by 1%) and seeing if any of the qualitative behavior changes.

f. (10 pts.) Place the moon at the L4 Lagrange point. (The notebook changes the `alleqn` function to set the initial conditions correctly.) Run the simulation. Does the Lagrange point appear to be stable? You are not looking for a proof here, just for qualitative observations to support your conclusion. (*Hint:* Try changing the initial conditions very slightly (e.g. by 1%) and seeing if any of the qualitative behavior changes.

Academic Honesty

You may use, without proof, any results from class or from your text by simply quoting the result and giving the reference (*e.g.* equation number or page number). You should understand *how* that result was obtained, but you need not transcribe the derivation.

If you get bogged down with any of the problems, do not hesitate to discuss them with me or with a fellow student. However, if you discuss a problem with *anyone* (besides me) you should acknowledge that collaboration.