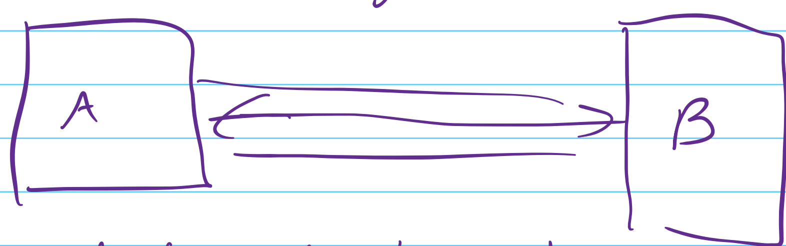
Chapter 2: The Second Law

Observation: There is a directionality to many processes. Heat flows from a hotter to a cooler, not the other way around. Why?

We will see this is matter of probability, but so high a probability it's effectively inevitable.

General Plan:

- Look at describing a single system's states



- Allow A to interact with B.
- See how states A + B evolve

3 core physical examples

1) Einstein solid
2) ideal gas
3) 2-state paramagnet (usually in problems)

2.1 Two-state systems

Cartoon system: coin flip. Two outcomes, "H", "T".

e.g. flip 4 coins: $2^4 = 16$ possibilities
some possible results

H H H T
H H T H
T T H H

Jargon:

Microstate: specify the state of each particle,
e.g. HHTH.

Macrostate: specify some macroscopic property, e.g. "3 Heads".

Multiplicity: # of microstates corresponding to one macrostate

$\Omega(3H) = 4$

list them:

H H H T
H H T H
H T H H
T H H H

$\Omega(\text{all}) = 2^4 = 16$ = total # of microstates.

probability of 3 heads in 4 tosses is then

$$P_{3H} = \frac{\Omega(3H)}{\Omega(\text{all})} = \frac{4}{16} = \frac{1}{4}$$

formula (see text for derivation)

N = Number of coins
n = # of heads

$$\Omega(N, n) = \frac{N!}{n!\,(N-n)!} = \binom{N}{n}$$

= "N choose n"

e.g. $\Omega(4,3) = \dfrac{4!}{3!\;1!} = \dfrac{4\cdot 3\cdot 2\cdot 1}{(3\cdot 2\cdot 1)(1)} = 4$ ✓

Interpret: There are 4 ways to pick the first H
3 " " " 2nd H
2 " " " 3rd H

but those 3 H's could be arranged in 6 different order $(6 = 3! = 3\cdot 2)$ so we need to divide the # of combinations by 6 if we are only interested in the macrostate: 3H's.

$$\therefore \Omega(4,3) = \frac{4\cdot 3\cdot 2}{(3\cdot 2)} = \frac{(4\cdot 3\cdot 2\cdot 1)}{3!\quad 1!}$$

$$\Omega(4,3) = \frac{4!}{3!\,(4-3)!} = 4.$$

See problem 2.1, 2.2 to scale it up.

2.3 is for HW.

2-state paramagnet

$$\uparrow\uparrow\downarrow\uparrow\downarrow\downarrow\ \downarrow\uparrow\ \uparrow$$

$N_\uparrow$ = # of spin up dipoles

$N_\downarrow$ = # of spin down dipoles

$$N = N_\uparrow + N_\downarrow$$

Magnetic energy in an external field $\propto N_\uparrow - N_\downarrow$

$$\Omega(N_\uparrow) = \binom{N}{N_\uparrow} = \frac{N!}{N_\uparrow!\,(N-N_\uparrow)!}$$

$$\Omega(N_\uparrow) = \frac{N!}{N_\uparrow!\,N_\downarrow!}$$