Problem 2.5

In[∘]:=

Problem 2.5. For an Einstein solid with each of the following values of N and q, list all of the possible microstates, count them, and verify formula 2.9.

(a) $N = 3$, $q = 4$

(b) $N = 3$, $q = 5$

(c) $N = 3$, $q = 6$

(d) $N = 4$, $q = 2$

(e) $N = 4$, $q = 3$

(f) $N = 1$, $q =$ anything

(g) $N =$ anything, $q = 1$

Einstein solid with ‘n’ oscillators and ‘q’ energy units:

In[∘]:=

$$\Omega[n\_,\ q\_] := \frac{(q + n - 1)!}{q!\,(n - 1)!}$$

We can also enumerate all the microstates. The Tuples function will list all possible orderings of ‘N’ integers running from 0 to ‘q’. We then Select[ ] only those that have a total energy of ‘q’. There are many other ways to do this in Mathematica.

```
In[∘]:= Tuples[Range[0, 4], 3] // TableForm;
```

```
In[∘]:= enumerate[n_, q_] := Select[Tuples[Range[0, q], n], Total[#] == q &];
```

a. N = 3, q = 4

```
In[∘]:= n = 3; q = 4;
```

In[ ]:=
```
microstates = enumerate[n, q];
TableForm[microstates]
{Ω[n, q], Length[microstates]}
```

Out[ ]//TableForm=

0	0	4
0	1	3
0	2	2
0	3	1
0	4	0
1	0	3
1	1	2
1	2	1
1	3	0
2	0	2
2	1	1
2	2	0
3	0	1
3	1	0
4	0	0

Out[ ]=

{15, 15}

b. N = 3, q = 5

In[ ]:=
```
n = 3; q = 5;
microstates = enumerate[n, q];
TableForm[microstates]
{Ω[n, q], Length[microstates]}
Clear[n, q]
```

Out[ ]//TableForm=

0	0	5
0	1	4
0	2	3
0	3	2
0	4	1
0	5	0
1	0	4
1	1	3
1	2	2
1	3	1
1	4	0
2	0	3
2	1	2
2	2	1
2	3	0
3	0	2
3	1	1
3	2	0
4	0	1
4	1	0
5	0	0

Out[ ]=
```
{21, 21}
```

c. N = 3, q = 6

In[ ]:=
```
n = 3; q = 6;
microstates = enumerate[n, q];
TableForm[microstates]
{Ω[n, q], Length[microstates]}
Clear[n, q]
```

Out[ ]//TableForm=

0	0	6
0	1	5
0	2	4
0	3	3
0	4	2
0	5	1
0	6	0
1	0	5
1	1	4
1	2	3
1	3	2
1	4	1
1	5	0
2	0	4
2	1	3
2	2	2
2	3	1
2	4	0
3	0	3
3	1	2
3	2	1
3	3	0
4	0	2
4	1	1
4	2	0
5	0	1
5	1	0
6	0	0

Out[ ]=

{28, 28}

d. N = 4, q = 2

In[ ]:=
```
n = 4; q = 2;
microstates = enumerate[n, q];
TableForm[microstates]
{Ω[n, q], Length[microstates]}
Clear[n, q]
```

Out[ ]//TableForm=

0	0	0	2
0	0	1	1
0	0	2	0
0	1	0	1
0	1	1	0
0	2	0	0
1	0	0	1
1	0	1	0
1	1	0	0
2	0	0	0

Out[ ]=

```
{10, 10}
```

e. N = 4, q = 3

```
In[ ]:= n = 4; q = 3;
microstates = enumerate[n, q];
TableForm[microstates]
{Ω[n, q], Length[microstates]}
Clear[n, q]
```

Out[ ]//TableForm=

0	0	0	3
0	0	1	2
0	0	2	1
0	0	3	0
0	1	0	2
0	1	1	1
0	1	2	0
0	2	0	1
0	2	1	0
0	3	0	0
1	0	0	2
1	0	1	1
1	0	2	0
1	1	0	1
1	1	1	0
1	2	0	0
2	0	0	1
2	0	1	0
2	1	0	0
3	0	0	0

Out[ ]=

```
{20, 20}
```

f. N = 1, q = anything

```
In[ ]:= Ω[1, q]
```

Out[ ]=

```
1
```

With only 1 oscillator, and a total energy of ‘q’, there is only one microstate: That oscillator must have energy ‘q’.

g. N = anything, q = 1.

```
In[ ]:= Ω[n, 1]
```

Out[ ]=

$$\frac{n!}{(-1+n)!}$$

In[ ]:=
```
FullSimplify[%, Element[n, Integers]]
```

Out[ ]=

n

With only one energy unit, it could be in any of the ‘n’ oscillators.