

Problem 2.23. Consider a two-state paramagnet with 10^{23} elementary dipoles, with the total energy fixed at zero so that exactly half the dipoles point up and half point down.

- How many microstates are "accessible" to this system?
- Suppose that the microstate of this system changes a billion times per second. How many microstates will it explore in ten billion years (the age of the universe)?
- Is it correct to say that, if you wait long enough, a system will eventually be found in every "accessible" microstate? Explain your answer, and discuss the meaning of the word "accessible."

2-state paramagnet $\rightarrow N_{\downarrow} = N_{\uparrow} = \frac{1}{2}N$

Start: $\Omega = \frac{N!}{N_{\downarrow}! (N - N_{\downarrow})!} = \frac{N!}{((N/2)!)^2}$

Let $N \rightarrow \text{large}$, e.g. 10^{23}

$$\Omega(N, N/2) \approx \frac{N^N e^{-N} \sqrt{2\pi N}}{\left[(N/2)^{N/2} e^{-N/2} \sqrt{2\pi N/2} \right]^2}$$

$$= \frac{N^N e^{-N} \sqrt{2\pi N}}{(N/2)^N e^{-N} \pi N} = 2^N \sqrt{\frac{2}{\pi N}}$$

Note $2^N = \text{total \# of microstates}$ including all $N_{\downarrow}$ values. But 2^N is a very large #, while $\sqrt{\frac{2}{\pi N}}$ is just $1/\text{large}$ and can be neglected.

OR.... The multiplicity function is very sharply peaked so that nearly all the states have $N_1 = 1/2 N$.

How big is $2^{10^{23}}$?

$$\ln 2^{10^{23}} = 10^{23} \ln 2$$

(b) Explore 10^9 states/second for
 $\tau = \frac{86400 \times 365.25}{1 \text{ yr}} = 3.1 \times 10^7 \text{ yr}$

$$M_{\text{states}} = \frac{10^9 \text{ states}}{\text{s}} \times \frac{3.1 \times 10^7 \text{ s}}{\text{yr}} \times 10 \times 10^9 \text{ yr}$$

$$= 3.1 \times 10^{25} \text{ states}$$

Compare M_{states} to Ω

$$\frac{M_{\text{states}}}{\Omega} \sim \frac{3 \times 10^{25}}{2^{10^{23}}} \sim 0$$

no appreciable fraction of states are accessed.

