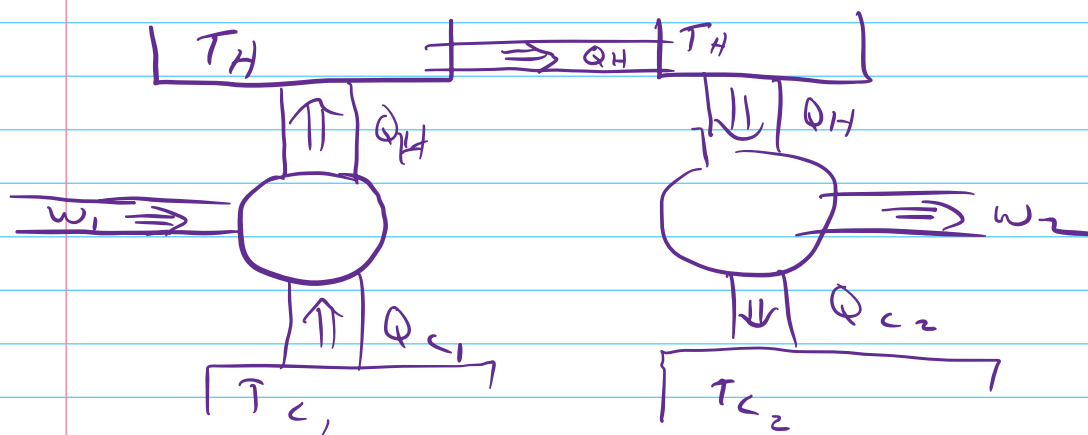


**Problem 4.17.** Prove that *if* you had a refrigerator whose COP was better than the ideal value (4.9), you could hook it up to an ordinary Carnot engine to make an engine that produces no waste heat.



Question: Since  $Q_H > W_1$ , can you use  $Q_H$  to drive a heat engine and get  $W_2 > W_1$ ? (Assume both devices are ideal)

$$Q_H = W_1 + Q_{C1} \Rightarrow W_2 = W_1 + (Q_{C1} - Q_{C2})$$

So  $W_2 > W_1$  if  $Q_{C1} > Q_{C2}$ .

$$2^{nd} \text{ law: } \frac{Q_H}{T_H} = \frac{Q_{C1}}{T_{C1}} = \frac{Q_{C2}}{T_{C2}}$$

$$\text{or } Q_{C1} = \frac{T_{C1}}{T_{C2}} Q_{C2}$$

$$\text{So } Q_{C1} > Q_{C2} \Rightarrow T_{C1} > T_{C2}$$

So yes, you can use the exhaust heat from device 1 to drive device 2, but you can't connect the cold reservoirs since heat won't flow from 2 to 1.

The net effect is you do input work  $W_1$  and input heat  $Q_{C1}$  in and get output work  $W_2$  and waste heat  $Q_{C2}$ .

$$e_{net} = \frac{W_2 - W_1}{Q_{c1}}$$

$$W_1 = \frac{Q_{c1}}{COP_1}$$

$$W_2 = e_2 Q_H = e_2 (W_1 + Q_{c1})$$

$$W_2 = e_2 \left( \frac{Q_{c1}}{COP_1} + Q_{c1} \right)$$

$$W_2 = e_2 Q_{c1} \left( \frac{1}{COP_1} + 1 \right)$$

$$e_{net} = \frac{e_2 Q_{c1} \left( \frac{1}{COP_1} + 1 \right) - \frac{Q_{c1}}{COP_1}}{Q_{c1}}$$

$$e_{net} = \frac{e_2}{COP_1} + e_2 - \frac{1}{COP_1} = e_2 \left( 1 + \frac{1}{COP_1} \right) - \frac{1}{COP_1}$$

Plug in ideal formulas for  $e_2$  and  $COP_1$

$$e_{net} = \frac{T_H - T_{c2}}{T_H} \left( 1 + \frac{T_H - T_{c1}}{T_{c1}} \right) - \frac{T_H - T_{c1}}{T_{c1}}$$

$$\begin{aligned} e_{net} &= \frac{T_H - T_{c2}}{T_H} + \frac{(T_H - T_{c1})(T_H - T_{c2})}{T_H T_{c1}} - \frac{T_H - T_{c1}}{T_{c1}} \\ &= 1 - \frac{T_{c2}}{T_H} + \frac{T_H^2}{T_H T_{c1}} - \frac{T_{c1} T_H}{T_H T_{c1}} - \frac{T_H T_{c2}}{T_H T_{c1}} + \frac{T_{c1} T_{c2}}{T_H T_{c1}} \\ &\quad - \frac{T_H}{T_{c1}} + 1 \end{aligned}$$

$$e_{\text{net}} = 2 - \frac{T_{C2}}{T_H} + \frac{T_H}{T_{C1}} - 1 - \frac{T_{C2}}{T_{C1}} + \frac{T_{C2}}{T_H} - \frac{T_H}{T_{C1}}$$

$$e_{\text{net}} = 1 - \frac{T_{C2}}{T_{C1}}$$

Note this is exactly what you'd expect for a heat engine that takes heat  $Q_{C1}$  as input and generates work  $(W_2 - W_1)$  and exhausts heat  $Q_{C2}$ .