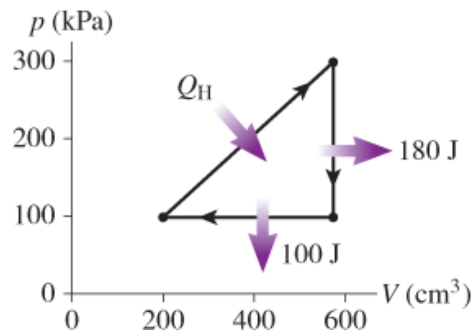
103. ||| **INT** What are (a) the heat Q_H extracted from the hot reservoir and (b) the efficiency for a heat engine described by the pV diagram of Figure P12.103?

Given: Net Work = 40J for one full cycle.

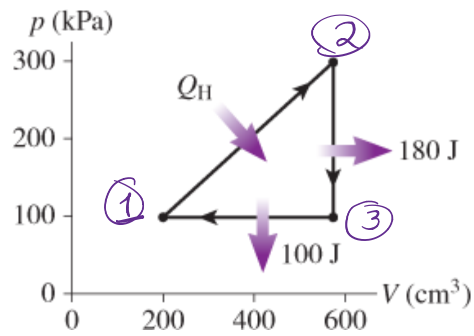
Figure P12.103 Hint: Use the 1st law for one full cycle.



103. ||| **INT** What are (a) the heat Q_H extracted from the hot reservoir and (b) the efficiency for a heat engine described by the pV diagram of Figure P12.103?

Given: Net work = 40 J for a full cycle.

Figure P12.103 Hint: Use the first law for a complete cycle.



1st Law: $Q_{total} = \Delta E_{th,total} + W_{total}$

$$Q_{total} = \frac{3}{2} nR(T_1 - T_1) + 40\text{ J}$$

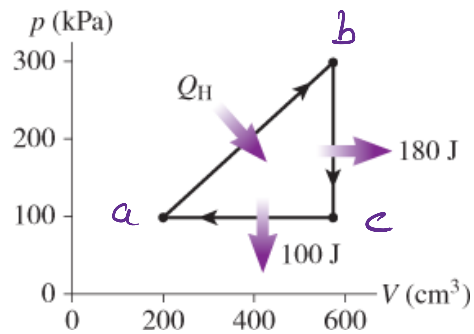
$$Q_H - 180\text{ J} - 100\text{ J} = 0 + 40\text{ J}$$

$$Q_H = 320\text{ J}$$

$$e = \frac{W}{Q_H} = \frac{40\text{ J}}{320\text{ J}} = \frac{1}{8} = 12.5\%$$

103. ||| INT What are (a) the heat Q_H extracted from the hot reservoir and (b) the efficiency for a heat engine described by the pV diagram of Figure P12.103?

Figure P12.103



Here is a longer solution, working through each process step.

$a \rightarrow b$ 1st Law $Q_H = \Delta E_{th} + W_{ab}$

what is ΔE_{th}? $\Delta E_{th} = \frac{3}{2} N k_B \Delta T = \frac{3}{2} n R \Delta T$

what is ΔT_{ab}? $p_a V_a = nRT_a$

$p_b V_b = nRT_b$

$\Delta T_{ab} = T_b - T_a = \frac{1}{nR}(p_b V_b - p_a V_a)$

$\Delta E_{th} = \frac{3}{2} nR \, \Delta T_{ab} = \frac{3}{2}(p_b V_b - p_a V_a)$

$\Delta E_{th} = \frac{3}{2}\left((300 \times 10^3\,\text{Pa})(600 \times 10^{-6}\,\text{m}^3) - (100 \times 10^3\,\text{Pa})(200 \times 10^{-6}\,\text{m}^3)\right)$

$\Delta E_{th} = 240\,\text{J}$

W_{ab} = ? (This is harder because it's not constant p)

W_{ab} is the area under the curve = (area of triangle) + area of rectangle below

$W_{ab} = \frac{1}{2}(\Delta V (p_b - p_a)) + (p_a \cdot \Delta V)$

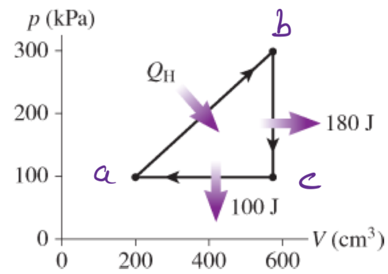
$= \frac{1}{2}(p_b + p_a)(\Delta V) = (200 \times 10^3\,\text{Pa})(400 \times 10^{-6}\,\text{m}^3)$

$W_{ab} = 80\,\text{J}$

$\therefore$ $a \rightarrow b$ summary

$Q_H = W_{ab} + \Delta E_{th}$

$= 80\,\text{J} + 240\,\text{J} = 320\,\text{J}$



(b) Efficiency $= e = \frac{W}{Q_H}$

what is net work? the area inside the triangle: $W = \frac{1}{2}(P_b - P_a)(V_b - V_c)$

$= \frac{1}{2}(200 \times 10^3 Pa)(400 \times 10^{-6} m^3) = 40 J$

$e = \frac{40 J}{320 J} = \frac{1}{8} \approx 0.125 = \boxed{12.5\%}$

Shortcut: 1^{st} law for full process:

$Q_{total} = \Delta E_{th, total} + W_{total}$

$Q_H - 180 J - 100 J = \underbrace{0}_{\text{complete cycle}} + \underbrace{40 J}_{\frac{1}{2}(\Delta P)(\Delta V)}$

$Q_H = 180 + 100 + 40 J = \boxed{320 J}$