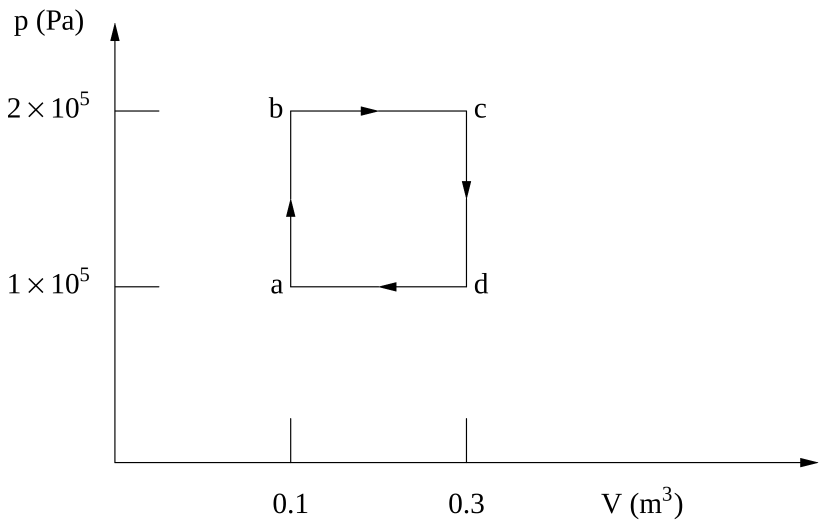
3. (60 pts.) A 6-mole sample of a monatomic ideal gas is taken through the cycle shown in the figure.



a. (20 pts.) Find the net work done during the cycle a -> b -> c -> d -> a.

b. (10 pts.) What is the heat absorbed by the system during the process a -> b?

c. (10 pts.) What is the heat absorbed by the system during the process b -> c?

d. (5 pts.) The efficiency is the net work (part a) divided by the total heat input into the system (parts b + c). What is the efficiency for this system?

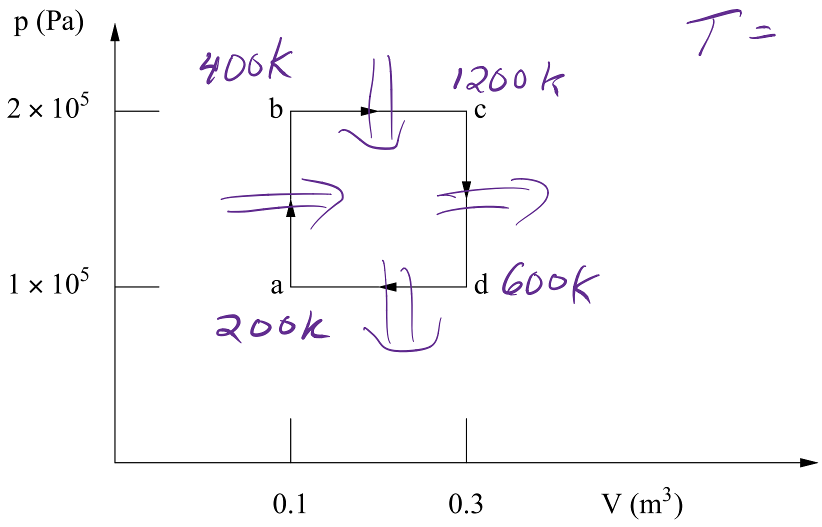
e. (5 pts.) What is the ratio of the coldest temperature T_a to the hottest temperature T_c encountered during this cycle?

f. (10 pts.) What is the efficiency of a Carnot engine operating between those two extreme temperatures? Does the process in this problem violate the second law of thermodynamics?

3. (60 pts.) A 6-mole sample of a monatomic ideal gas is taken through the cycle shown in the figure.

$pV = nRT$

$T = \frac{pV}{nR}$



$W = p\Delta V$

a. (20 pts.) Find the net work done during the cycle a -> b -> c -> d -> a.

$W_{a\to b} = p(\Delta V) = 0$

$W_{b\to c} = p_b(V_c - V_b) = \left(2.0\times10^5 \frac{N}{m^2}\right)(0.3m^3 - 0.1m^3)$
$= 4 \times 10^4 J$

$W_{c\to d} = 0$

$W_{d\to a} = p_a(V_a - V_d) = (1.0\times10^5)(0.1 - 0.3) J$
$= -2\times10^4 J$

$W_{net} = W_{ab} + W_{bc} + W_{cd} + W_{da} = \boxed{+2.0 \times 10^4 J}$

b. (10 pts.) What is the heat absorbed by the system during the process a -> b?

$Q_{a\to b} = \Delta E_{th\,a\to b} + W_{a\to b} = \Delta E_{th\,a\to b}$

$\Delta E_{th} = \frac{3}{2} N k_B \Delta T_{a\to b} = \frac{3}{2} n R \Delta T_{a\to b}$

$Q_{a\to b} = \frac{3}{2}(6\ moles)\left(8.314 \frac{J}{mol\cdot K}\right)(400 - 200K)$

$Q_{a\to b} = 15{,}000\ J$

c. (10 pts.) What is the heat absorbed by the system during the process b -> c?

$$Q_{b\to c} = W_{bc} + \Delta E_{th\ b-c}$$

$$Q_{b\to c} = W_{bc} + \frac{3}{2} nR(T_c - T_b)$$

$$= 40{,}000\ \text{J} + \frac{3}{2}(6\ \text{mol})\left(8.314 \frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(800\ \text{K})$$

$$Q_{b\to c} = 100{,}000\ \text{J}$$

d. (5 pts.) The efficiency is the net work (part a) divided by the total heat input into the system (parts b + c). What is the efficiency for this system?

$$e = \frac{W}{Q_{in}} = \frac{20{,}000\ \text{J}}{115{,}000\ \text{J}} = 0.17 \approx 17\%$$

e. (5 pts.) What is the ratio of the coldest temperature T_a to the hottest temperature T_c encountered during this cycle?

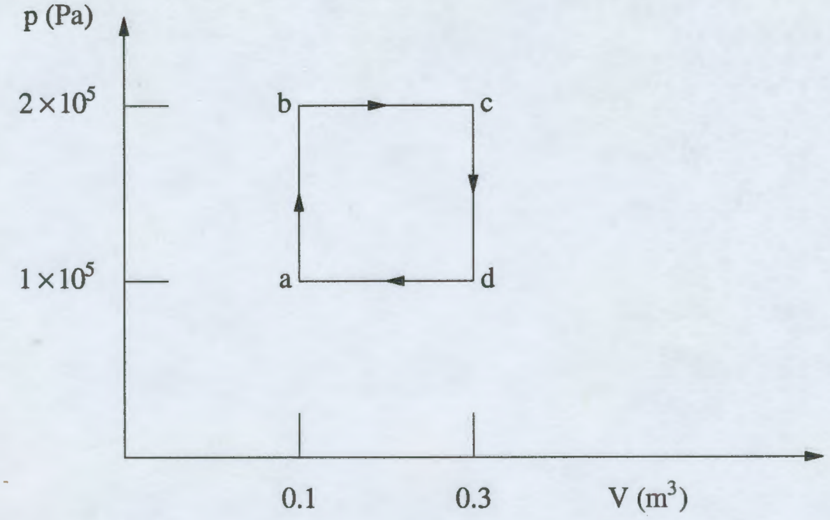
$$e_{max} = 1 - \frac{T_c}{T_H} = 1 - \frac{200\text{k}}{1200\text{k}} = \frac{5}{6}$$

$$e_{max} \approx 83\%$$

$\therefore$ This does not violate the 2nd law.

f. (10 pts.) What is the efficiency of a Carnot engine operating between those two extreme temperatures? Does the process in this problem violate the second law of thermodynamics?

3. (60 pts.) A 6-mole sample of a monatomic ideal gas is taken through the cycle shown in the figure.



a. (20 pts.) Find the net work done during the cycle a -> b -> c -> d → a .

$W_{a\to b} = 0$

$W_{b\to c} = (2\times10^5)(0.3-0.1) = 4\times10^4\ \text{J}$

$W_{c\to d} = 0$

$W_{d\to a} = (1\times10^5)(0.1-0.3) = -2\times10^4\ \text{J}$

Net work = $+\boxed{2\times10^4\ \text{J}}$

b. (10 pts.) What is the heat absorbed by the system during the process a -> b?

$Q_{ab} = W_{ab} + \Delta U_{ab} = 0 + \Delta U_{ab} = \Delta U_{ab}$

$Q_{ab} = nC_v\Delta T = nC_v(T_b - T_a) = nC_v\left(\frac{p_bV_b}{nR} - \frac{p_aV_a}{nR}\right)$

Note $C_v = \frac{3}{2}R$

$Q_{ab} = n\left(\frac{3}{2}R\right)\cdot\frac{V_a}{nR}(p_b - p_a) = \frac{3}{2}V_a(\Delta p) = \boxed{15{,}000\ \text{J}}$

c. (10 pts.) What is the heat absorbed by the system during the process b -> c? $Q_{bc} = W_{bc} + \Delta U_{bc} = 40{,}000 + m C_v \Delta T$

$$Q_{bc} = 40{,}000 + m\left(\frac{3}{2}R\right)\left(\frac{p_b}{mR}\right)(\Delta V) =$$

$$= 40{,}000 + \frac{3}{2} p_b \Delta V = 40{,}000 + 60{,}000 = \boxed{10^5\ \text{J}}$$

d. (5 pts.) The efficiency is the net work (part a) divided by the total heat input into the system (parts b + c). What is the efficiency for this system?

$$e = \frac{2\times10^4}{115{,}000} = \boxed{0.174 = 17.4\%}$$

e. (5 pts.) What is the ratio of the coldest temperature T_a to the hottest temperature T_c encountered during this cycle?

$$\frac{T_a}{T_c} = \frac{p_aV_a/mR}{p_cV_b/mR} = \frac{p_aV_a}{p_cV_c} = \left(\frac{1}{2}\right)\left(\frac{0.1}{0.3}\right) = \frac{1}{6}.$$

[Aside: $T_a = 200\ \text{K}$, $T_c = 1203\ \text{K}$].

f. (10 pts.) What is the efficiency of a Carnot engine operating between those two extreme temperatures? Does the process in this problem violate the second law of thermodynamics?

$$e_{carnot} = 1 - \frac{T_{cold}}{T_{hot}} = 1 - \frac{1}{6} = 0.83 = 83\%$$

No, this does not violate the 2nd law: $e < e_{carnot}$, as it must be.