



Question: Since $Q_H > W_1$, can you use Q_H to drive a heat engine and get $W_2 > W_1$? (Assume both devices are ideal)

$$Q_H = W_1 + Q_{c1} \Rightarrow W_2 = W_1 + (Q_{c1} - Q_{c2})$$

$$Q_H = W_2 + Q_{c2}$$

So $W_2 > W_1$ if $Q_{c1} > Q_{c2}$.

2nd law: $\frac{Q_H}{T_H} = \frac{Q_{c1}}{T_{c1}} = \frac{Q_{c2}}{T_{c2}}$

or $Q_{c1} = \frac{T_{c1}}{T_{c2}} Q_{c2}$

So $Q_{c1} > Q_{c2} \Rightarrow T_{c1} > T_{c2}$

So yes, you can use the exhaust heat from device 1 to drive device 2, but you can't connect the cold reservoirs since heat won't flow from 2 to 1.

The net effect is you do input work W_1 and input heat Q_{c1} in and get output work W_2 and waste heat Q_{c2} .

$$e_{net} = \frac{W_2 - W_1}{Q_{c1}}$$

$$W_1 = \frac{Q_{c1}}{COP_1} \quad W_2 = e_2 Q_H = e_2 (W_1 + Q_{c1})$$

$$W_2 = e_2 \left(\frac{Q_{c1}}{COP_1} + Q_{c1} \right)$$

$$W_2 = e_2 Q_{c1} \left(\frac{1}{COP_1} + 1 \right)$$

$$e_{net} = \frac{e_2 Q_{c1} \left(\frac{1}{COP_1} + 1 \right) - \frac{Q_{c1}}{COP_1}}{Q_{c1}}$$

$$e_{net} = \frac{e_2}{COP_1} + e_2 - \frac{1}{COP_1} = e_2 \left(1 + \frac{1}{COP_1} \right) - \frac{1}{COP_1}$$

Plug in ideal formulas for e_2 and COP_1

$$e_{net} = \frac{T_H - T_{c2}}{T_H} \left(1 + \frac{T_H - T_{c1}}{T_{c1}} \right) - \frac{T_H - T_{c1}}{T_{c1}}$$

$$e_{net} = \frac{T_H - T_{c2}}{T_H} + \frac{(T_H - T_{c1})(T_H - T_{c2})}{T_H T_{c1}} - \frac{T_H - T_{c1}}{T_{c1}}$$

$$= 1 - \frac{T_{c2}}{T_H} + \frac{T_H^2}{T_H T_{c1}} - \frac{T_{c1} T_H}{T_H T_{c1}} - \frac{T_H T_{c2}}{T_H T_{c1}} + \frac{T_{c1} T_{c2}}{T_H T_{c1}}$$

$$= 1 - \frac{T_{c2}}{T_{c1}} + 1$$

$$e_{\text{net}} = 2 - \frac{T_{C2}}{T_H} + \frac{T_H}{T_{C1}} - 1 - \frac{T_{C2}}{T_{C1}} + \frac{T_{C2}}{T_H} - \frac{T_H}{T_{C1}}$$

$$e_{\text{net}} = 1 - \frac{T_{C2}}{T_{C1}}$$

Note this is exactly what you'd expect for a heat engine that takes heat Q_{C1} as input and generates work $(W_2 - W_1)$ and exhausts heat Q_{C2} .