Magnetic Susceptibility

1. (Recall from E&M, if you covered it....)
Response of a Linear Isotropic Homogeneous Medium to an applied magnetic field

$\vec{M} \equiv$ magnetic moment / volume
units: (current · area) / volume = A/m

$\vec{H} \equiv$ applied magnetic field (more in a moment)

LIHM: $\vec{M} = \chi \vec{H}$
Then net magnetic field is

$$\vec{B} = \mu_0 (\vec{M} + \vec{H})$$

(Indeed, this is the definition of $\vec{H} = \frac{1}{\mu_0} \vec{B} - \vec{M}$.)

Various materials:

$\chi > 0$ paramagnetic
dipoles tend to align with external field

$\chi < 0$ diamagnetic
dipoles (often easy to think of them as induced) tend to align <u>opposite</u> the applied field.

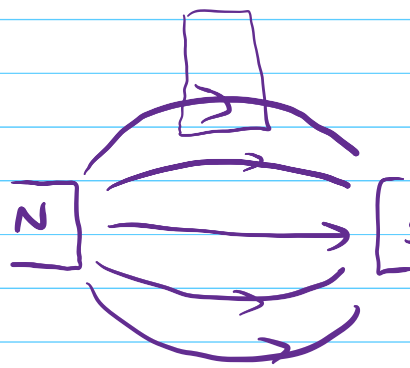
χ typically very small, $\sim 10^{-5}$.

Ferromagnetic: χ not constant. $\vec{M}$ can be non-zero even in the absence of an applied field.

What happens if you bring a paramagnetic sample close to a magnet?

Crude picture:

Sample is attracted to the magnet.
(more details below.)

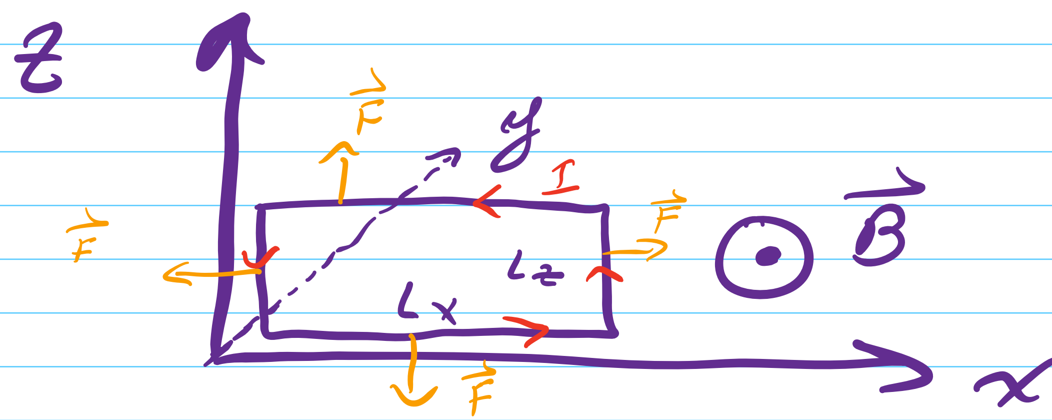


In our case, the sample ~~is~~ is fixed but tries to lift the magnet, so the measured mass of the magnet <u>decreases</u>.

Look more carefully:

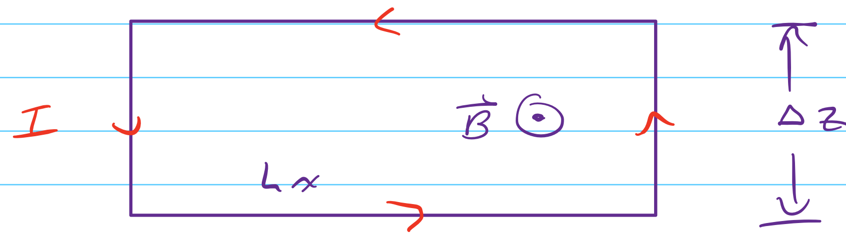
Force on a current loop (whose dipole moment is aligned with the external field):

a) in a uniform $\vec{B}$ field



Net $\vec{F} = 0$ in a uniform $\vec{B}$.
(Torque is not zero → current loop tends to align with $\vec{B}$.)

b) Current loop in a gradient
Suppose $\vec{B}$ is not uniform in z



$$\Delta F = I L_x B_{top} - I L_x B_{bot}$$
$$= I L_x \Delta B$$

Recall magnetic dipole moment $\mu \equiv I \cdot (\text{Area})$

$$\Delta F = I L_x \Delta z \left(\frac{\Delta B}{\Delta z}\right) = \mu \frac{\Delta B}{\Delta z}$$

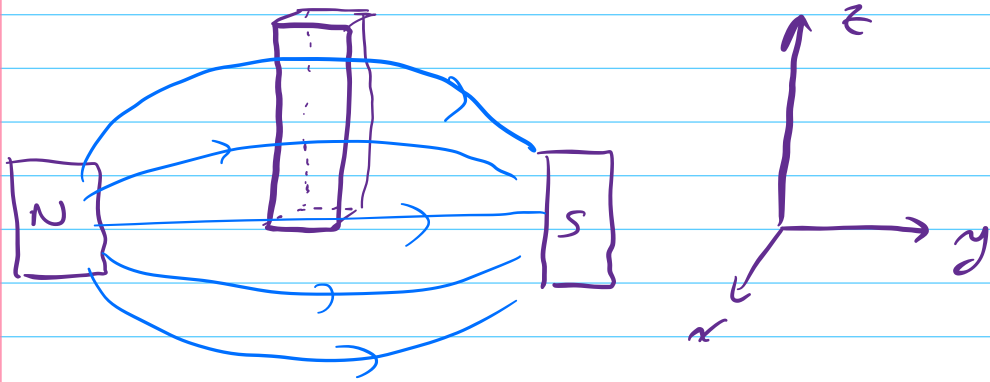
Now think of magnetization M. Sample has a cross-sectional area $A = L_x L_y$

$$M = \frac{\mu}{L_x L_y \Delta z} = \frac{\mu}{A \Delta z} \Rightarrow \mu = M A \Delta z$$

$\therefore$ $\Delta F = M A \Delta B$ for that small loop.
Recall $M = \chi H$
and $B = \mu_0 (M + H) = \mu_0 (1 + \chi) H$
Thus

average value over small loop.
$$\Delta F = \chi H A \mu_0 (1 + \chi) \Delta H$$

Now, integrate over a tall sample



$$F = \int dF = \chi A \mu_0 (1+\chi) \int H\,dH$$
$$= \chi A \mu_0 (1+\chi) \frac{1}{2} H^2 \Big|_{bottom}^{top}$$

Lastly measure H (by sticking a current-carrying wire between the magnets and measuring the force).

With no sample present, call it B

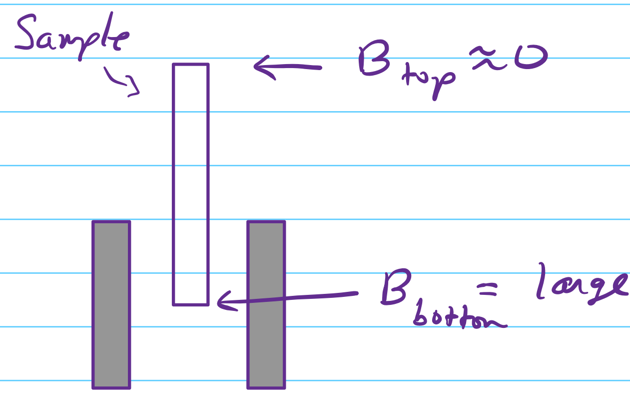
$$B = \mu_0 H \Rightarrow H = \frac{1}{\mu_0} B$$

Then

$$F = \chi A \mu_0 (1 + \chi) \frac{1}{2} \left(\frac{B}{\mu_0}\right)^2 \Big|_{bottom}^{top}$$

drop ($\sim 10^{-5}$)

$$\boxed{F = \frac{\chi A}{2\mu_0} \left(B_{top}^2 - B_{bottom}^2\right)}$$



If χ = positive (paramagnetic) the material is attracted into the region of strong field. Equivalently, the magnets are pulled up.

F shows up as a change in mass recorded by the balance

$$F = \Delta m\, g$$

$\therefore$

$$\Delta m = \frac{\chi A}{2\mu_0 g}\left(\underbrace{B_{top}^2}_{\text{should be } = 0.} - B_{bottom}^2\right)$$

$\therefore$ Guoy balance:

Measure $\Delta m \Rightarrow$ learn χ.

Predicting χ: Atomic or Solid State Physics. Requires quantum mechanics.

(Classically $\chi = 0$).

See Section 2 of the Teach Spin manual.