Electron Spin Resonance -- Uncertainty in B

In[14]:=
```
Clear["Global`*"]; DateString[]
```

Out[14]=
```
Tue 8 Oct 2024 10:47:01
```

There are two main sources of uncertainty in B (and hence in g):
1. Random: Each individual measurement of I_DC (and hence B_DC) has uncertainty. It can be challenging to find the exact current that gives resonance. Presumably, the value recorded will sometimes be too high and sometimes too low. These random fluctuations will be reflected in the uncertainty in the slope. Equivalently, we can look at the RMSE (Root Mean Square Error) or Sqrt[-fit["EstimatedVariance"]] in Mathematica.

2. Systematic: The magnetic field depends on the coil radius R and coil separation D. Uncertainties in R and D also affect B, but do so systematically. This will not show up in the fit statistics.

The formula for the magnetic field is

$$\mu 0\, n\, i\, \frac{r^2}{\left(r^2+(d/2)^2\right)^{3/2}}$$

Propagating the uncertainty is tedious here, but it is simple to just plug in two different numbers and see how big an effect there is. Since we are ultimately interested in a ratio (i.e. a percent error) we can just compute the percent error due to the geometric part of the formula.

In[15]:=
```
(* Measurements of R and D, in cm, and their uncertainties. *)
r0 = 7.08; δr = 0.05; d0 = 7.20; δd = 0.1;
```

In[16]:=
```
f[r_, d_] := r^2 / (r^2 + (d / 2)^2)^(3/2)
```

In[17]:=
```
pctdiff[δr_, δd_] := 100 * (f[r0 + δr, d0 + δd] - f[r0, d0]) / f[r0, d0]
```

In[18]:=
```
pctdiff[δr, δd]
```

Out[18]=
```
-1.11607
```

So these uncertainties correspond to about a 1.1% uncertainty in B. This should be compared to the statistical uncertainty in B from the graph.

Mathematica can also handle the uncertainties more directly with the 'Around' function.

In[19]:=
```
r = Around[r0, δr]
```

Out[19]=

7.08 ± 0.05

In[20]:=
```
d = Around[d0, δd]
```

Out[20]=

7.20 ± 0.10

In[21]:=
```
term = f[r, d]
```

Out[21]=

0.1000 ± 0.0024

In[22]:=
```
100 * term["Uncertainty"] / term["Value"]
```

Out[22]=

2.35828