Physics 133-02: Physics II—Electricity, Magnetism, and Waves
Diffraction Grating Problem

Problem 1: Light of a single wavelength is incident on a diffraction grating with 500 slits per mm. Several bright fringes are observed on a screen behind the grating, including one at 45.7° and one next to it at 72.6°. What is the wavelength of the light?

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First, note that the slit spacing $d = \frac{1}{500}$ mm. Although we know $d\sin\theta = m\lambda$, we are not given the specific m values. However, we are given that the two specified angles correspond to adjacent m values. Thus we know

$$\begin{aligned} d\sin 45.7° &= m\lambda \\ d\sin 72.6° &= (m+1)\lambda \end{aligned}$$

This is a set of two equations with two unknowns, m and λ.

The simplest approach is to simply subtract the two equations:

$$\begin{aligned} d\sin 72.6° &= (m+1)\lambda \\ d\sin 45.7° &= m\lambda \\ d\sin 72.6° - d\sin 45.7° &= \lambda \\ \frac{1}{500\,\text{mm}}\left(\sin 72.6°a - \sin 45.7°\right) &= \boxed{4.77\times10^{-4}\,\text{mm} = 477\,\text{nm}} \end{aligned}$$

Alternatively, since we know m is an integer, another viable approach is simply trial and error. Guess $m = 1$, use the first equation to compute λ, and then check if $(m+1) = 2$ works for the second equation. If not, then try again with $m = 2$, *etc.*

Try $m = 1$:

$$\begin{aligned} \lambda &= \frac{d\sin 45.7°}{m} = \frac{\frac{1}{500\,\text{mm}}\sin 45.7°}{1} = 1.431\times10^{-3}\,\text{mm} = 1431\,\text{nm} \\ \lambda &= \frac{d\sin 45.7°}{m+1} = \frac{\frac{1}{500\,\text{mm}}\sin 72.6°}{2} = 0.954\times10^{-3}\,\text{mm} = 954\,\text{nm} \end{aligned}$$

These do not agree, so the guess $m = 1$ does not work. Try $m = 2$.

$$\begin{aligned} \lambda &= \frac{d\sin 45.7°}{m} = \frac{\frac{1}{500\,\text{mm}}\sin 45.7°}{2} = 716\,\text{nm} \\ \lambda &= \frac{d\sin 45.7°}{m+1} = \frac{\frac{1}{500\,\text{mm}}\sin 72.6°}{3} = 636\,\text{nm} \end{aligned}$$

These also do not agree, so the guess $m = 2$ also does not work. Try $m = 3$.

$$\begin{aligned} \lambda &= \frac{d\sin 45.7°}{m} = \frac{\frac{1}{500\,\text{mm}}\sin 45.7°}{3} = 477\,\text{nm} \\ \lambda &= \frac{d\sin 45.7°}{m+1} = \frac{\frac{1}{500\,\text{mm}}\sin 72.6°}{4} = 477\,\text{nm} \end{aligned}$$

These do agree, so $\boxed{\lambda = 477\,\text{nm}}$.

This is in the range of visible light.

Finally, another approach is to consider ratios—consider the first equation divided by the second. The unknown value of λ cancels. The resulting ratio should be a recognizable ratio of small integers.

$$\begin{aligned}
\frac{\sin 45.7°}{\sin 72.6°} &= \frac{m}{m+1} \\
0.75 &= \frac{m}{m+1} \\
3 &= m
\end{aligned}$$

Then

$$\begin{aligned}
d \sin 45.7° &= 3\lambda \\
\lambda &= \frac{1}{3} \frac{1}{500\,\mathrm{mm}} \sin 45.7° \\
\lambda &= \boxed{4.77 \times 10^{-4}\,\mathrm{mm} = 477\,\mathrm{nm}}
\end{aligned}$$