

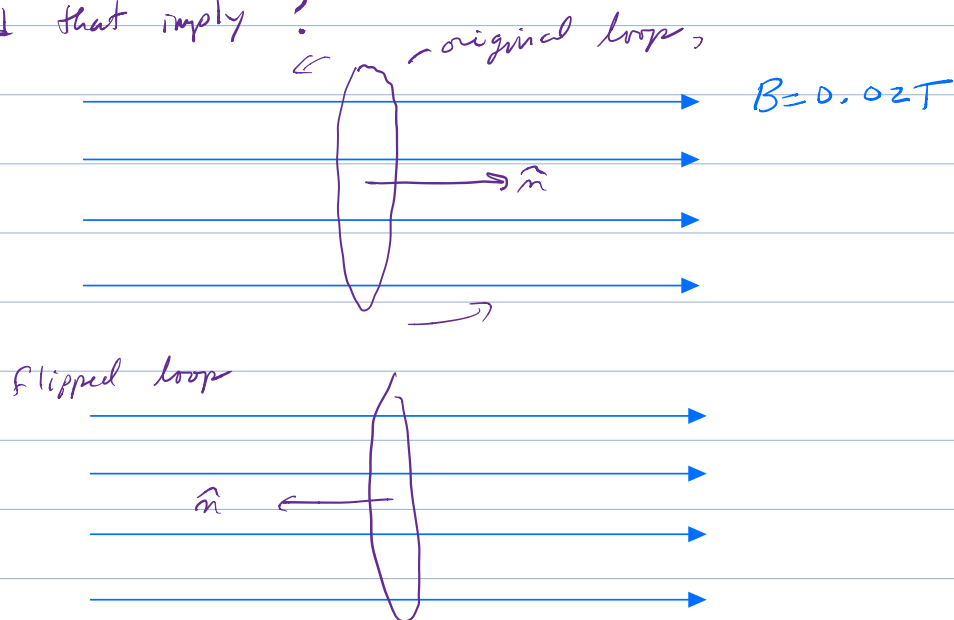
10. An electric generator consists of a rectangular coil of wire rotating about its longitudinal axis which is perpendicular to a magnetic field of 2.0×10^{-2} T. The coil measures 10.0 cm \times 20.0 cm and has 120 turns of wire. The ends of the wire are connected to an external circuit. At what speed (in rev/s) must you rotate this coil in order to induce an alternating emf of amplitude 12.0 V between the ends of the wire?

See the "Quantitative Prelecture Video"
for a more typical test problem.

Here is a similar simplification: How quickly
must you rotate from 0° to 180° to have an
average emf of 12.0 V? What rotation speed
would that imply?

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$$\text{Area} = 10\text{ cm} \times 20\text{ cm} = 0.10\text{ m} \times 0.2\text{ m} = 0.02\text{ m}^2$$

$$N = 120\text{ turns}$$

$$\begin{aligned}\text{original flux: } \Phi_i &= NBA \cos 0^\circ \\ &= (120)(0.02\text{ T})(0.02\text{ m}^2)(1)\end{aligned}$$

$$\Phi_i = 0.048\text{ T m}^2$$

$$\begin{aligned}\text{final flux: } \Phi_f &= NBA \cos 180^\circ \\ &= (120)(0.02\text{ T})(0.02\text{ m}^2)(-1)\end{aligned}$$

$$\Phi_f = -0.048\text{ T m}^2$$

$$\text{Faraday's Law: } \mathcal{E} = -\frac{\Delta\Phi}{\Delta t}$$

$$\mathcal{E} = -\frac{(\Phi_f - \Phi_i)}{\Delta t} = \frac{0.096 \text{ Tm}^2}{\Delta t}$$

want $\mathcal{E} = 12.0 \text{ V}$

$$\therefore \Delta t = \frac{0.096 \text{ Tm}^2}{12.0 \text{ V}} = 0.008 \text{ s}$$

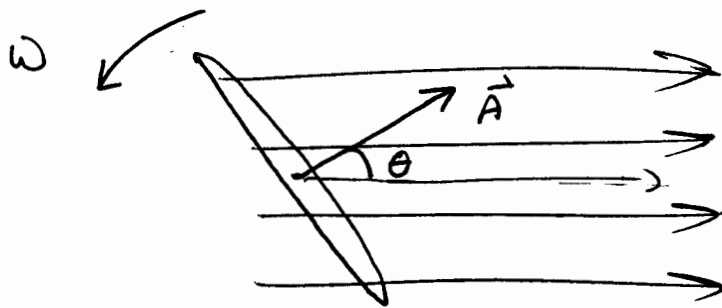
This is $\frac{1}{2} T$ for a full rotation

$$\therefore T = 0.016 \text{ s}$$

$$f = 1/T = 62.5 \text{ Hz}$$

(Calculus version below gives 39.8 Hz, so the assumption that derivative is approximated by the average change is OK, but not great.)

e.g. generator. (#10)



uniform
 $\vec{B} = 2 \times 10^{-2} \text{ T } \hat{x}$

Loop, area $0.02 \text{ m}^2 = A$

$N = 120$ turns.

want $\mathcal{E}_{\text{max}} = 120 \text{ V}$

frequency of rotation $f = ?$

$$\Phi_B = N \int \vec{B} \cdot d\vec{A} = N \int B dA \cos \theta = N B \cos \theta \int dA$$

$$\Phi_B = NBA \cos \theta$$

$$\mathcal{E} = - \frac{d\Phi_B}{dt} = - NBA \sin \theta \left(\frac{d\theta}{dt} \right)$$

$\frac{d\theta}{dt} = \omega = \text{angular velocity}$

Derivatives won't be on our test!

$$= - NBA \sin(\omega t) \cdot \omega$$

$$\mathcal{E} = - NBA 2\pi f \sin(\omega t)$$

oscillates -1 to $+1$

~~want~~ $\mathcal{E}_{\text{max}} = NBA 2\pi f$

want $\mathcal{E}_{\text{max}} = 120$, $12.0 = (120)(0.02)(0.02) 2\pi f$
 $39.8 \text{ Hz} = f$

