Problem 3: (20 pts.) In some chain molecules, electrons behave as if they were trapped in a one-dimensional box of length L. The allowed wavelengths for such an electron are given by the same formula as the allowed wavelengths for a vibrating string clamped at both ends.

Consider such a box of length $L = 1.2\,\text{nm}$. An electron is in the $n = 3$ state such that $L = \frac{3}{2}\lambda$.

a. (5 pts.) What is the corresponding momentum p?

b. (5 pts.) What is the corresponding kinetic energy E_3?

c. (5 pts.) Suppose the electron now makes a transition to a new state with $n = 2$ such that that $L = \frac{2}{2}\lambda$. What is the new kinetic energy E_2?

d. (5 pts.) What is the wavelength of the photon emitted in the transition from E_3 to E_2?

Problem 3: (20 pts.) In some chain molecules, electrons behave as if they were trapped in a one-dimensional box of length L. The allowed wavelengths for such an electron are given by the same formula as the allowed wavelengths for a vibrating string clamped at both ends.

Consider such a box of length $L = 1.2\,\text{nm}$. An electron is in the $n = 3$ state such that $L = \frac{3}{2}\lambda$.

a. (5 pts.) What is the corresponding momentum p?

$p = \frac{h}{\lambda}$ $\lambda = \frac{2}{3}L = 0.8\,nm$

$p = \frac{6.626 \times 10^{-34}\,J\cdot s}{0.8 \times 10^{-9}\,m} = 8.28 \times 10^{-25}\,kg\,m/s$

b. (5 pts.) What is the corresponding kinetic energy E_3?

$E_3 = \frac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{(8.28 \times 10^{-25}\,kg\,m)^2}{2(9.11 \times 10^{-31}\,kg)} = 3.76 \times 10^{-19}\,J$

$(2.35\,eV)$

c. (5 pts.) Suppose the electron now makes a transition to a new state with $n = 2$ such that that $L = \frac{2}{2}\lambda$. What is the new kinetic energy E_2?

$p = \frac{h}{\lambda} = \frac{6.626 \times 10^{-34}\,J\cdot s}{1.2\,nm} = 5.52 \times 10^{-25}\,kg\,m/s$

$E_2 = \frac{p^2}{2m} = 1.673 \times 10^{-19}\,J$ $(1.04\,eV)$

d. (5 pts.) What is the wavelength of the photon emitted in the transition from E_3 to E_2?

$\frac{hc}{\lambda} = \Delta E \Rightarrow \lambda = \frac{hc}{\Delta E} = \frac{(6.626 \times 10^{-34})(3 \times 10^{8})}{3.76 \times 10^{-19} - 1.673 \times 10^{-19}}\,m$

$\lambda = 950\,nm$