Physics 112: General Physics II: Electricity, Magnetism, and Optics
Beta Decay

Problem 30.67 The technique known as potassium-argon dating is used to date volcanic rock and ash, and thus establish dates for nearby fossils, like this 1.8-million-year-old hominid skull. The potassium isotope ^{40}K decays with a 1.28-billion-year half-life and is naturally present at very low levels. The most common decay mode is beta-minus decay into the stable isotope ^{40}Ca, but 10.9% of decays result in the stable isotope ^{40}Ar. The high temperatures in volcanoes drive argon out of solidifying rock and ash, so there is no argon in newly formed material. After formation, argon produced in the decay of ^{40}K is trapped, so ^{40}Ar builds up steadily over time. Accurate dating is possible by measuring the ratio of the number of atoms of ^{40}Ar and ^{40}K.

a. What type of decay results in ^{40}Ar?

Putting in the explicit proton numbers Z, $^{40}_{19}$K changes into $^{40}_{18}$Ar. Effectively, a proton has been replaced by a neutron, so this is electron capture (or inverse beta decay).

$$^{40}_{19}\mathrm{K} + \mathrm{e}^- \rightarrow {}^{40}_{18}\mathrm{Ar} + \nu_e$$

b. How much energy is released in this reaction? The mass of ^{40}K is 39.963 998 475 u, and the mass of ^{40}Ar is 39.962 383 123 u.

Taking the difference of the masses:

$$\begin{aligned}
\Delta m &= m_{^{40}\mathrm{K}} - m_{^{40}\mathrm{Ar}} \\
\Delta m &= 39.963\,998\,475\,\mathrm{u} \\
&\quad - 39.962\,383\,123\,\mathrm{u} \\
\Delta m &= \ \ 0.001\,615\,\mathrm{u} \\
\Delta E &= (\Delta m)c^2 = (0.001\,615) \times (931.5\,\mathrm{MeV}) = \boxed{1.50\,\mathrm{MeV}}
\end{aligned}$$

c. 1.8 million years after its formation, what fraction of the ^{40}K initially present in a sample has decayed?

Let the intial number of ^{40}K atoms be N_0. (Since all the questions ask for fractions or ratios, this number will never actually be needed.) The number of potassium atoms left after a time $t = 1.80 \times 10^6$ yr is N_K.

$$N_\mathrm{K} = N_0 \left(\frac{1}{2}\right)^{t/t_{1/2}}$$

The number that have decayed is then $N_0 - N_k$, and the fraction that has decayed is

$$N_0 - N_\mathrm{K} = N_0 \left(1 - \left(\frac{1}{2}\right)^{t/t_{1/2}}\right)$$
$$\frac{N_0 - N_\mathrm{K}}{N_0} = \left(1 - \left(\frac{1}{2}\right)^{t/t_{1/2}}\right)$$
$$\frac{N_0 - N_\mathrm{K}}{N_0} = \left(1 - \left(\frac{1}{2}\right)^{(1.80\times 10^6\ \mathrm{yr})/(1.28\times 10^9\ \mathrm{yr})}\right)$$
$$\frac{N_0 - N_\mathrm{K}}{N_0} = (1 - 0.9990) = 0.0009743$$

d. 1.8 million years after its formation, what is the ^{40}Ar/^{40}K ratio of the sample?

Only 10.9% of those decays result in ^{40}Ar. The remainder result in ^{40}Ca. Thus the number of ^{40}Ar atoms is

$$N_\mathrm{Ar} = (0.109) \times (N_0 - N_\mathrm{K})$$

The problem asks for ratios, so divide both sides by N_K.

$$\frac{N_\mathrm{Ar}}{N_\mathrm{K}} = (0.109) \times \left(\frac{N_0}{N_\mathrm{K}} - 1\right)$$

But note that we found above that $N_\mathrm{K}/N_0 = 0.9990$, so

$$\frac{N_\mathrm{Ar}}{N_\mathrm{K}} = (0.109) \times \left(\frac{1}{0.9990} - 1\right) = 0.0001063$$