Chapter 17: Wave Optics

17.1: What is Light? Several different ways of viewing light, depending on context.

Ray Model. (Ch, 18). Wavelength << size of objects, e.g. light through a window.

Wave Model (Ch. 17) Wavelength ~ size of objects, e.g. sound through a window, or light through a very narrow slit.

Photon Model (Ch. 28) Quantum realm

The wave model is relevant when wavelength of wave is roughly the same size as any obstacles or apertures. That is the subject of this chapter.

$$v = \lambda f$$

In vacuum, $v = c = 3.00 \times 10^8 \mathrm{m/s}$

(slower in a medium — more in section 17.4.)

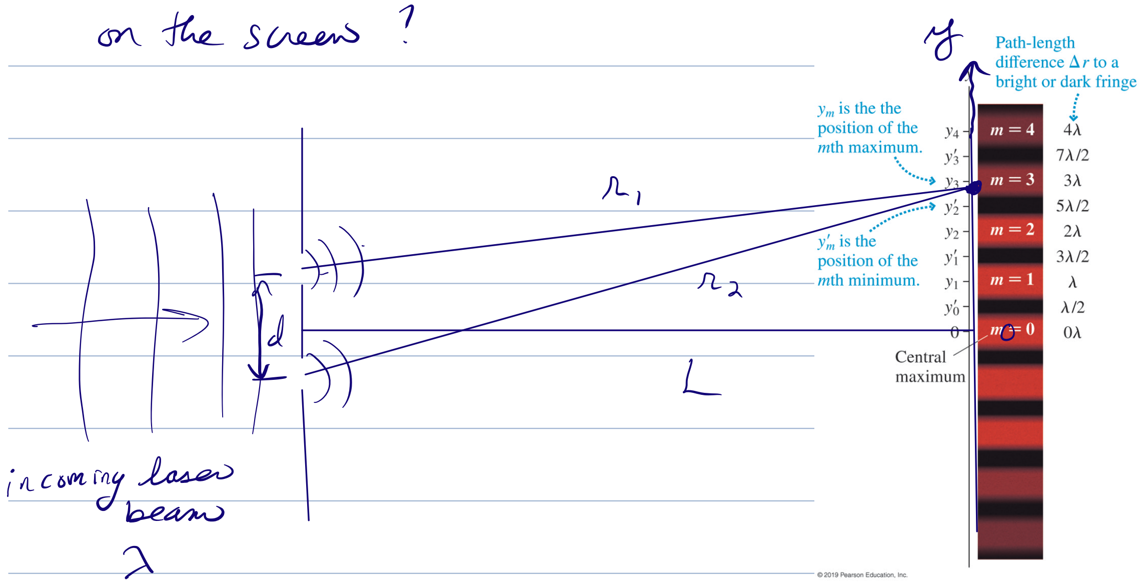
17.2 The Interference of Light

Double-slit interference

Just as with our two speakers problems, two light sources can interfere.

Require: 1) same wavelength (or frequency)
2) coherent sources (usually get by splitting the original source in two.)

e.g. laser strikes 2 slits. What do you see on the screen?



You don't just see 2 spots — you see a series of bright and dark spots.

Constructive interference: $\Delta r = m\lambda$

Destructive interference: $\Delta r = (m + 1/2)\lambda$

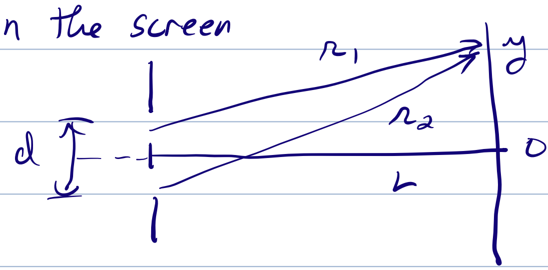
How to calculate Δr?

e.g. $\lambda = 632.8\ nm = 632.8 \times 10^{-6}\ mm$ (Red HeNe laser)

$d = 0.1\ mm$

$L = 2.000\ m = 2\,000\ mm$

Way #1: Brute force. Consider a point "y" on the screen



$$r_1 = \sqrt{L^2 + (y - d/2)^2}$$
$$r_2 = \sqrt{L^2 + (y + d/2)^2}$$

Then, you will get constructive interference (a bright spot) when $\Delta r = r_2 - r_1 = m\lambda$.

e.g. look at $y = 12.656300\ mm$

$r_1 \simeq 2000.0397290\ mm$ (Why all the digits? The Δr is so tiny!)

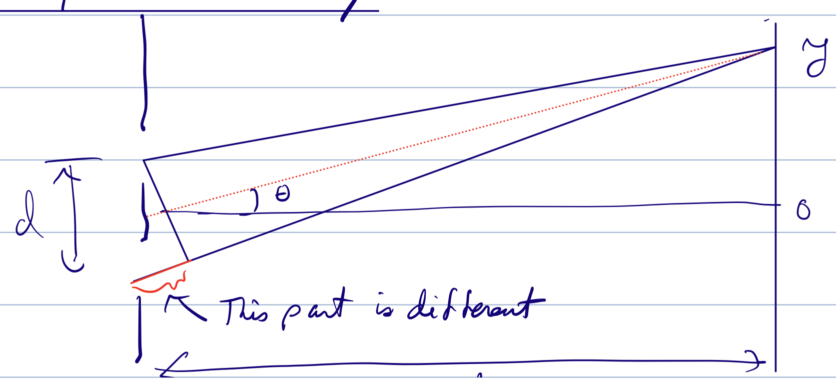
$r_2 \simeq 2000.0403618\ mm$

$\Delta r = .0006328\ mm = 632.8\ nm$

Since $\Delta r = 1\lambda$, this will be a bright spot.

Look for a better way to get Δr.

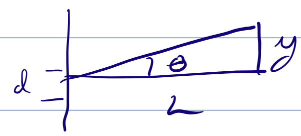
Way #2: Geometry



To an excellent approximation

$$\Delta r = d \sin\theta \text{ , where}$$

$$\sin\theta = \frac{y}{\sqrt{L^2+y^2}} .$$



So for our numbers above

$y = 12.6536 \text{ mm}$ $\quad L = 2{,}000 \text{ mm}$

$\theta = \tan^{-1}(y/L) = 0.36257°$

$\Delta r = d\sin\theta = (0.1\text{mm})(\sin(0.36257°))$

$\Delta r = 0.0006328 \text{mm} = 632.8 \text{nm}$ ✓

$\therefore$

Maxima (bright spots)	$d\sin\theta = m\lambda$
Minima (dark spots)	$d\sin\theta = (m+\tfrac{1}{2})\lambda$

e.g. $L = 2.000\text{m} = 2{,}000\text{mm}$

$\lambda = 632.8\text{nm} = 632.8\times10^{-6}\text{mm}$

$d = 0.1\text{mm}$

Where are the bright spots?

$m = 0.$ center. $\Delta r = 0.$

$m = 1$ $\quad \Delta r = d\sin\theta_1 = 1\lambda$

$\sin\theta_1 = 1\lambda/d = 0.006328$

$\theta_1 = \sin^{-1}(1\lambda/d) = 0.36257°$

$\tan\theta_1 = y_1/L$

$y_1 = L\tan\theta_1 = 12.6563 \text{ mm}$

$m = 2$ $\quad d \sin\theta_2 = 2\lambda$

$$\sin\theta_2 = 2\lambda/d$$

$$\theta_2 = 0.725155$$

$$y_2 = L\tan\theta_2 = 25.314\text{ mm}$$

Where are the dark spots?

$$\Delta r = d\sin\theta = (m + 1/2)\lambda$$

$m = 0$: $\quad d\sin\theta_0 = 1/2\,\lambda$

$$\sin\theta_0 = \tfrac{1}{2}\lambda/d \Rightarrow \theta_0 = \sin^{-1}\left(\lambda/2d\right)$$

$$y_0 = L\tan\theta_0 = 6.328\text{ mm}$$

$m = 1$ $\quad d\sin\theta_1 = (1 + 1/2\,\lambda_0)$

$$\theta_1 = \sin^{-1}\left(\tfrac{3}{2}\lambda_0/d\right)$$

$$y_1 = L\tan\theta_1 = 18.985\text{ mm}.$$

back to picture .... .

One last simplification: If the angles are small,

then

$$\sin\theta = \frac{y}{\sqrt{L^2+y^2}} \approx \frac{y}{L} = \tan\theta$$

so

Bright spots: $\quad d\sin\theta = m\lambda$ becomes

$$d\,\frac{y_m}{L} = m\lambda \Rightarrow y_m = m\frac{L\lambda}{d}$$

Dark spots: $\quad y_m = (m+\tfrac{1}{2})\,L\,\dfrac{\lambda}{d}.$

Qualitative observations:

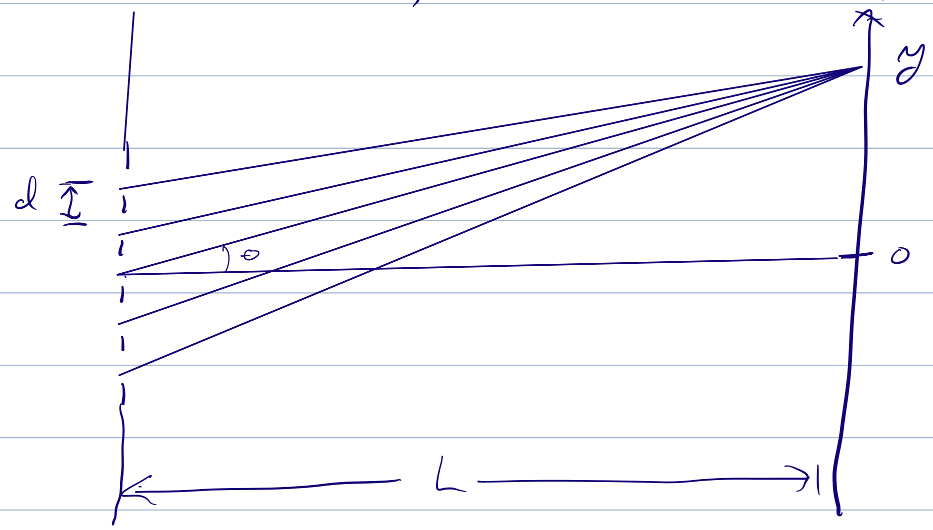
1) Larger $d \Rightarrow$ smaller y

Smaller $d \Rightarrow$ larger y

(demo – Mathematica). (example: Ch17-two-slit).

17.3 The diffraction grating

Use more than 2 slits, each a distance d apart



when $d \sin\theta = m\lambda$, each successive pair of slits reinforces to give constructive interference. The bright spots get sharper.

Still true: $\boxed{\text{Bright spots @ } d\sin\theta = m\lambda}$

Diffraction grating = large # of slits. Usually give $1/d$, e.g. $\frac{500 \text{ slits}}{\text{mm}} \Rightarrow d = \frac{1\,\text{mm}}{500} = 0.002\,\text{mm}$

→ Small d ⇒ large angles. Don't use small angle approximation.

→ Maximum angle is 90°.

→ Different λ's ⇒ different angles for same m, i.e. spreads out colors.

See examples.

Next: single slit diffraction