20.4 The electric field of multiple charges

For multiple charges, we use superposition:

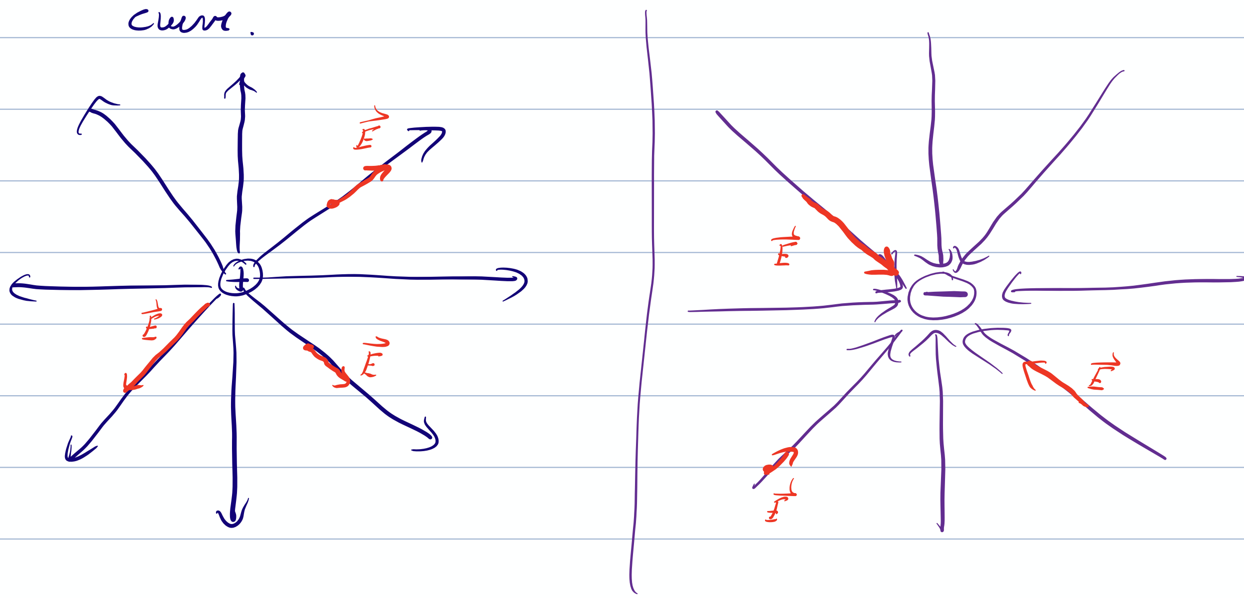
$\vec{E} = \vec{E}_1 + \vec{E}_2 + \cdots$

we already saw this in the previous segment.

Two ideas here

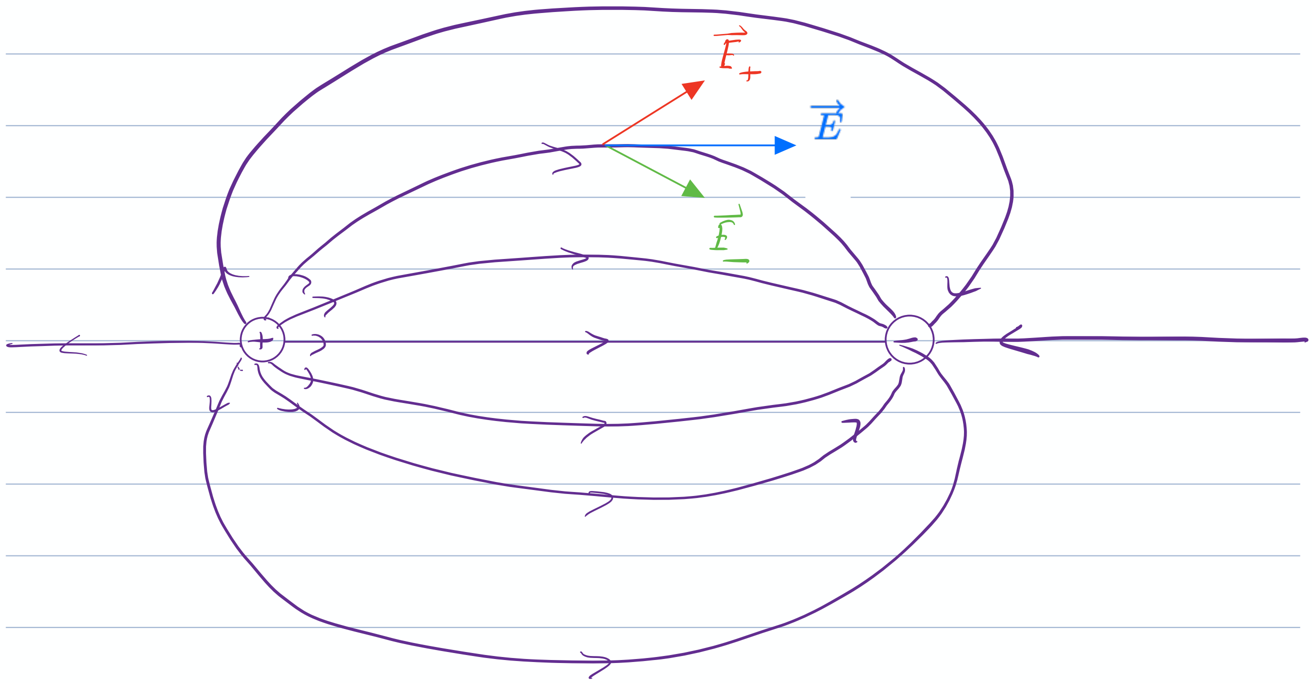
1) Electric Field Line diagrams
2) Uniform Fields

Electric Field Lines a pictoral tool to aid in visualization. Instead of individual vectors, draw smooth curves where $\vec{E}$ is tangent to the curve.



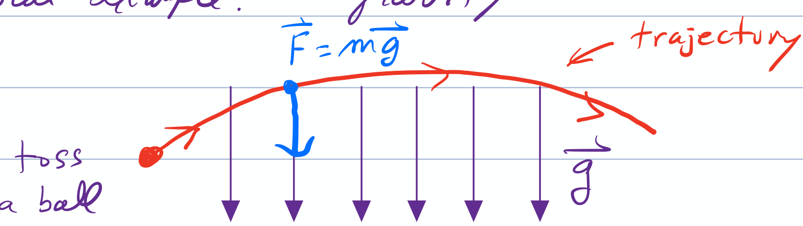
- Lines start on + charges and end on − charges.
- At any point $\vec{E}$ is tangent to the field line.
- The density of lines is related to the strength of $\vec{E}$.

Dipole (opposite charges separated by a distance L.)



These lines show the direction of $\vec{E}$ (and hence $\vec{F} = q_0\vec{E}$ that a particle would experience.) They are not necessarily the trajectories particles would follow. Recall $\vec{F} = m\vec{a}$ gives the acceleration, not the position or velocity.

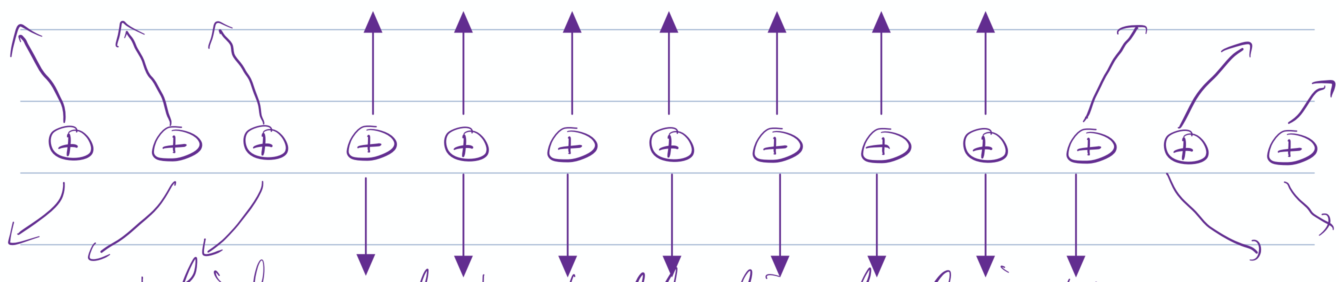
Trivial example: gravity



at any point, the field line shows the direction of the force.

Uniform electric field

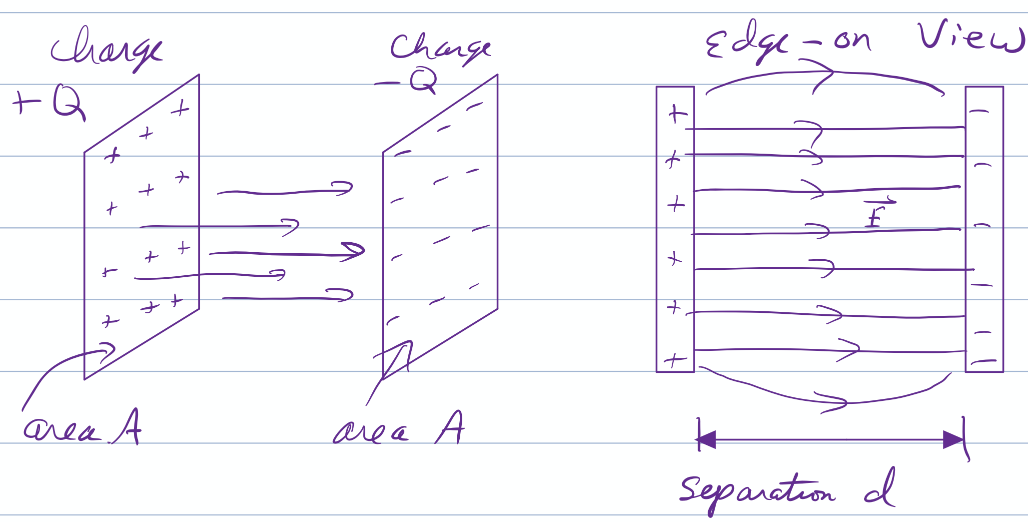
Consider a long line of charges:



Which way do the field lines bend in the middle? There is some "fringing" on the edges, but far from the edges, the field points straight away from the line.

Plane of charge $\Longrightarrow$ uniform Field
(both direction and magnitude are the same everywhere).

Parallel Plate Capacitor



If d is "small" compared to the size of the plates,
Between the plates, away from the edges,

$E \simeq$ uniform

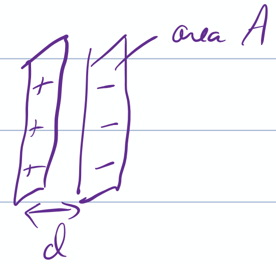
$\vec{E}$ points from $(+)$ to $(-)$

$$E = \frac{1}{\epsilon_0}\frac{Q}{A} = \frac{\sigma}{\epsilon_0}$$

$$\epsilon_0 = \frac{1}{4\pi k} \simeq 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$$

$\frac{Q}{A}$ = surface charge density $= \sigma$, $\frac{C}{m^2}$.

e.g. two square plates, 10 cm on a side, separated by 5 mm. Put $Q = 1.6 \times 10^{-10}$ C on one plate, $-Q$ on the other plate. What is $\vec{E}$ between the plates?



$A = 0.1\,m \times 0.1\,m = 0.01\,m^2$

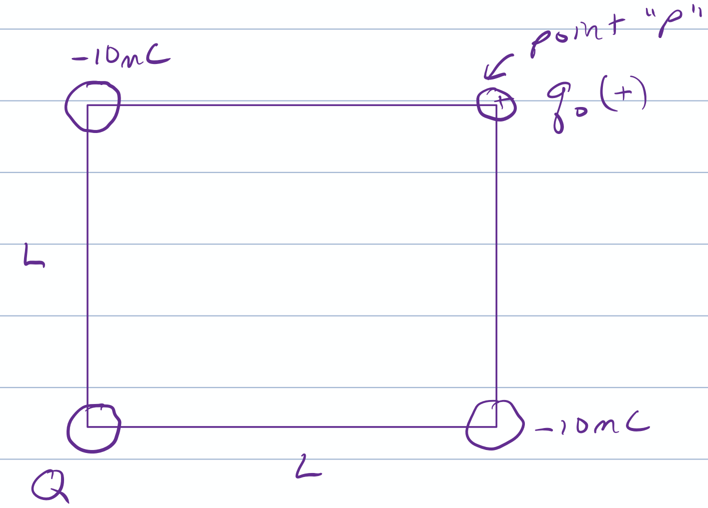
$Q = 1.6 \times 10^{-10}$ C

$\sigma = \frac{Q}{A} = 1.6 \times 10^{-8}\, C/m^2$

$$E = \frac{\sigma}{\epsilon_0} = \frac{1.6 \times 10^{-8}\, C/m^2}{8.85 \times 10^{-12}\, C^2/N\cdot m^2} = 1810 \frac{N}{C}$$

(We didn't use d other than to note it is "small.")

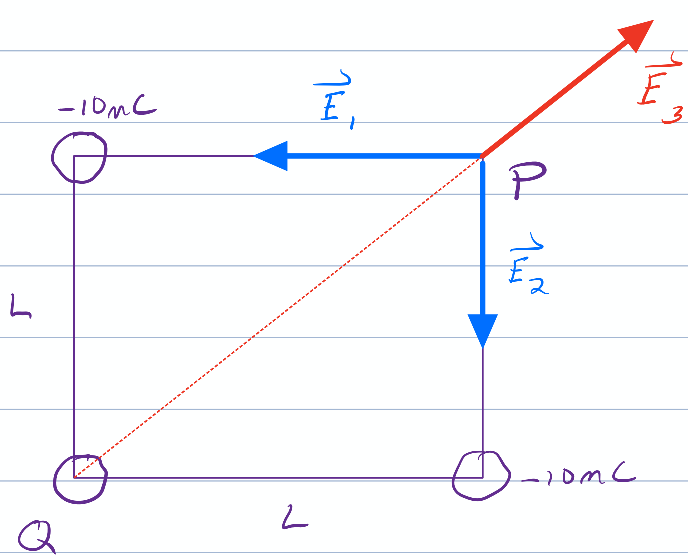
e.g. 20.61



What value of Q will make the force on $q_0 = 0$?

1st simplification. The charge q_0 really doesn't matter. If the electric field at point P is $\vec{E}$, then the force $\vec{F}_0$ on q_0 is $\vec{F}_0 = q_0 \vec{E}$. If $\vec{F}_0 = 0$, Then $\vec{E} = 0$.

$\therefore$ solve for what value of Q makes $\vec{E} = 0$ at the top corner.



Want $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = 0$.

magnitudes: $E_1 = \dfrac{K(10\,mC)}{L^2} = E_2$

$$E_3 = \frac{KQ}{(\sqrt{2}\,L)^2} = \frac{KQ}{2L^2}$$

$\vec{E}$ is a vector. Look at x and y components

$$E_x = E_1 \cos(180°) + E_2 \cos(-90°) + E_3 \cos 45°$$

$$0 = -E_1 + 0 + \frac{\sqrt{2}}{2} E_3$$

$$E_1 = \frac{\sqrt{2}}{2} E_3$$

$$\frac{K(10\,mC)}{L^2} = \frac{\sqrt{2}}{2} \frac{KQ}{2L^2}$$

$$Q = \frac{4 \cdot (10\,mC)}{\sqrt{2}} \approx \boxed{28.2\,mC}$$

Sign? Q must be positive to cancel out the attractions due to the other two charges.

symmetry $\Rightarrow$ get the same result for y-components.