Ch. 28 Part 2
Matter Waves

28.4: Matter Waves

We have seen light has properties of both a particle and a wave. Is the reverse true? Can a particle (such as an electron, proton, *etc.*) have properties of both a particle and a wave?

deBroglie (1924) proposed: *Yes!*. The same relations

$$p = \frac{h}{\lambda}\ , \text{ or } \lambda = \frac{h}{p}$$

hold for *both* particles and waves.

$p =$ momentum, $h = 6.63 \times 10^{-34}\,\mathrm{J\,s} =$ Planck's constant.

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- If p is of any appreciable size, as for a macroscopic object, then λ is tiny. If λ is less than the size of the object, it's not obvious it makes any sense.

28.4: Matter Waves

Implications:

1. Interference and Diffraction
2. Energy Quantization
3. Uncertainty

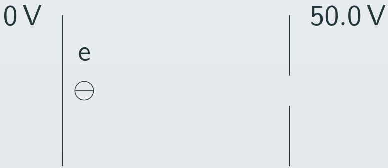
Electron Diffraction

Suppose you accelerate an electron through a potential difference of $\Delta V = 50.0\,\text{V}$. What is the wavelength of the electron?

Plan:

1. Find the speed v from the energy
2. Find the momentum from $p = mv$.
3. Find the wavelength from $\lambda = \dfrac{h}{p}$.

Electron Diffraction



$$E_i = E_f$$
$$K_i + U_i = K_f + U_f$$
$$0 + (-e)V_i = \frac{1}{2}mv_f^2 + (-e)V_f$$
$$0 + (-e)(V_i - V_f) = \frac{1}{2}mv_f^2$$

$$v_f = \sqrt{\frac{2e\Delta V}{m}} = \sqrt{\frac{2 \times (1.60 \times 10^{-19}\,\mathrm{C}) \times (50.0\,\mathrm{V})}{9.11 \times 10^{-31}\,\mathrm{kg}}} = 4.19 \times 10^{6}\,\mathrm{m/s}$$
$$p = mv = (9.11 \times 10^{-31}\,\mathrm{kg}) \times (4.19 \times 10^{6}\,\mathrm{m/s}) = 3.82 \times 10^{-24}\,\mathrm{kg\,m/s}$$
$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}\,\mathrm{J\,s}}{3.82 \times 10^{-24}\,\mathrm{kg\,m/s}} = \boxed{0.173\,\mathrm{nm}}$$

Electron Diffraction

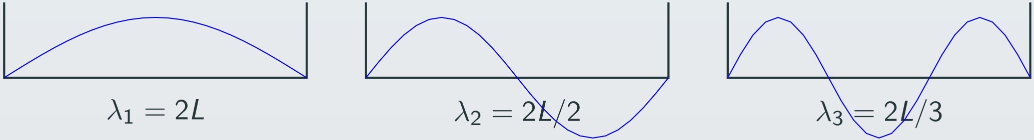
What can you do with such short waves? Send them through slits of various sorts and observe interference and diffraction. Note this wavelength is simlar to what we calculated for a typical X-ray, but with an energy of only 50 eV, rather than the 7150 eV we calculate for a 0.173 nm X-ray photon.

If we think of particles as having wave properties, then our work with standing waves suggests that if the wave is confined to a certain length L, then only certain wavelengths λ will be observed.

A Particle in a One-Dimensional Box

Consider a particle confined to a one-dimensional box of length L. Assume that the "wave" has to be zero at each end. What are the allowed wavelengths?

28.5: Energy is Quantized



Only certain λ values are allowed $\implies$ only certain $p = h/\lambda$ values will be observed $\implies$ only certain $E = \dfrac{p^2}{2m}$ values will be observed. (More details in a moment.)

28.6: Energy Levels and Quantum Jumps

Two basic ideas:

1. There are certain allowed energy “states”
2. Transitions between those states can be accompanied by the absorption or emission of a photon:

$$\frac{hc}{\lambda} = |\Delta E|$$

Example: Suppose there are two states: $E_i = 4.00\,\text{eV}$ and $E_f = 1.00\,\text{eV}$. A transition between the two is accompanied by the emission or absorption of a photon of energy 3.00 eV, and wavelength

$$\lambda = \left|\frac{hc}{\Delta E}\right| = \frac{1240\,\text{eV nm}}{3.00\,\text{eV}} = 413\,\text{nm}$$

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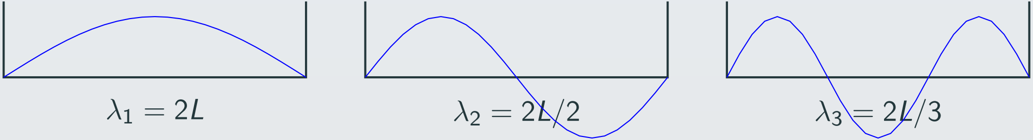
3. Corollary: Measuring wavelength λ can tell about available energy states.

Particle in a Box Example

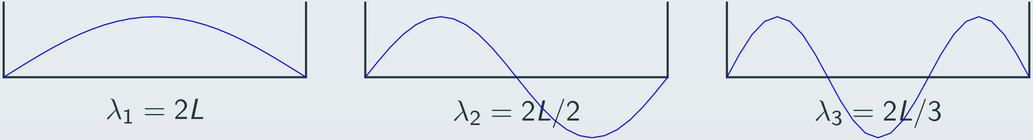
Suppose we confine an electron (mass $m = 9.11 \times 10^{-31}$ kg) to a box of length $L = 0.800\,\text{nm}$. Consider the 3 lowest-energy states. What are the allowed wavelengths, energies, and photon energies?



The allowed wavelengths are given by the usual standing wave condition $L = (integer) * \frac{\lambda_n}{2} \implies \lambda_n = \frac{2L}{n}$:

$$\lambda_1 = \frac{2L}{1} = 1.60\,\text{nm} \qquad \lambda_2 = \frac{2L}{2} = 0.800\,\text{nm} \qquad \lambda_3 = \frac{2L}{3} = 0.533\,\text{nm}$$

Particle in a Box Example



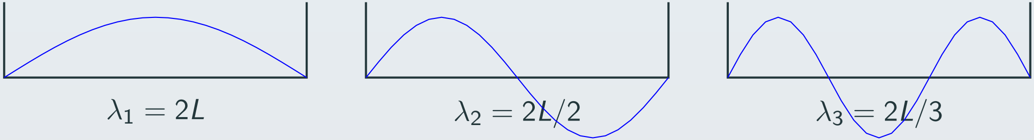
The corresponding momenta are given by deBroglie's relation $p_n = \frac{h}{\lambda_n}$.

$$p_1 = 4.14 \times 10^{-25}\,\mathrm{kg\,m/s} \quad p_2 = 8.28 \times 10^{-25}\,\mathrm{kg\,m/s} \quad p_3 = 1.24 \times 10^{-24}\,\mathrm{kg\,m/s}$$

Lastly, the kinetic energies are given by

$$E_n = \frac{1}{2} m v_n^2 = \frac{1}{2}\frac{p_n^2}{m} = \frac{p_n^2}{2m}$$

Particle in a Box Example



The resulting energies are given by $E_n = \frac{p_n^2}{2m}$.

$$E_1 = 9.41 \times 10^{-20}\ \text{J} \qquad E_2 = 3.77 \times 10^{-19}\ \text{J} \qquad E_3 = 8.47 \times 10^{-19}\ \text{J}$$

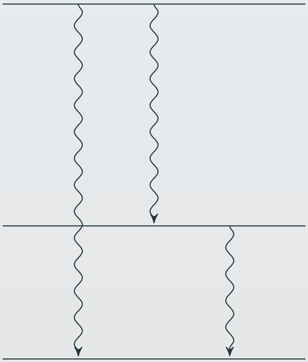
Converting to electron volts:

$$E_1 = 0.588\,\text{eV} \qquad E_2 = 2.35\,\text{eV} \qquad E_3 = 5.29\,\text{eV}$$

It is useful to draw an energy level diagram:

Particle in a Box Example

Draw the different energy levels as horizontal lines. Transitions between states are accompanied by photons, which are indicated as wavy lines in the diagram.

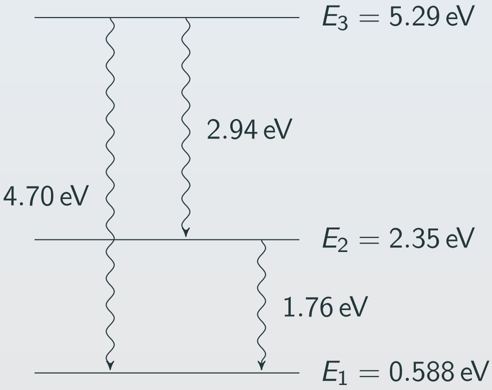


$E_3 = 5.29\,\text{eV}$

$E_2 = 2.35\,\text{eV}$

$E_1 = 0.588\,\text{eV}$

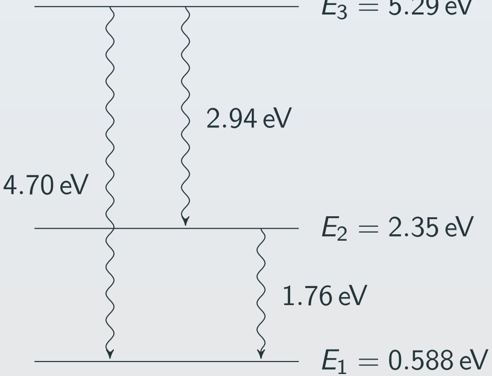
Particle in a Box Example



Each transition will be accompanied by a photon. The transition from state $3 \to 2$ involves a change in energy $\Delta E_{32} = 5.29\,\mathrm{eV} - 2.35\,\mathrm{eV} = 2.94\,\mathrm{eV}$. This corresponds to a photon of wavelength

$$\lambda_{32} = \frac{hc}{\Delta E_{32}} = \frac{1240\,\mathrm{eV\,nm}}{2.94\,\mathrm{eV}} = 422\,\mathrm{nm}$$

Particle in a Box Example



Each transition will be accompanied by a photon. The largest energy jump is accompanied by the shortest wavelength photon.

$$\lambda_{3\to 1} = \frac{hc}{\Delta E_{31}} = \frac{1240\,\text{eV nm}}{4.70\,\text{eV}} = 264\,\text{nm}$$

$$\lambda_{3\to 2} = \frac{hc}{\Delta E_{32}} = \frac{1240\,\text{eV nm}}{2.94\,\text{eV}} = 422\,\text{nm}$$

$$\lambda_{2\to 1} = \frac{hc}{\Delta E_{21}} = \frac{1240\,\text{eV nm}}{1.76\,\text{eV}} = 703\,\text{nm}$$

Be alert that these are the wavelengths of photons emitted in the transitions between states; they are not the wavelengths of the electron in those different states.

Formal Results

For a particle of mass m confined to a one-dimensional box of size L, the allowed energy states are given by the following set of steps:

$$\lambda_n = \frac{2L}{n}$$
$$p_n = \frac{h}{\lambda_n} = \frac{nh}{2L}$$
$$E_n = \frac{p_n^2}{2m} = n^2 \frac{h^2}{8mL^2}$$

Transitions between two levels (e.g. $n_i \to n_f$) involve the emission or absorption of a photon of wavelength

$$\lambda_{\text{photon}} = \left| \frac{hc}{E_f - E_i} \right|$$

Overall Results

- Get reasonable scale of answers for atomic spectra for little effort.
- Do see quantization
- Doesn't actually describe hydrogen (or any other atomic spectra) quantitatively
- Only 1-dimensional. Real atoms are 3-dimensional, and not simple boxes.
- Still, not too bad for such a simple model.

What's Next?

- More examples and applications
- Uncertainty
- Chapter 29: Move on to atomic energy levels.