# Ch. 30 Part 3
# Nuclear Decay and Half-Lives

# 30.5: Nuclear Decay and Half-Lives

Suppose you have a sample containing $N$ unstable radioactive nuclei. In a short time interval $\Delta t$, some number $\Delta N$ will decay. Define the activity $R$ by

$$R = \frac{\Delta N}{\Delta t} = \text{Number of decays per second.}$$

Observationally, $R$ is proportional to $N$. That is, on average, a specific fraction of the nuclei present decay in each time interval. Numerically,

$$R = \frac{N}{\tau}$$

where $\tau$ is called the "lifetime." It is different for different nuclei, and ranges from fractions of a second to billions of years.

# 30.5: Nuclear Decay and Half-Lives

Mathematically, this leads to

$$\frac{dN}{dt} = \frac{N}{\tau}$$

$$N = N_0 e^{-t/\tau}$$

$$R = R_0 e^{-t/\tau}$$

where $N_0$ is the number of nuclei at time $t = 0$, and $R_0$ is the activity at time 0.

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Units:

$$1\,\text{Becquerel} = 1\,\text{decay/s} = 1\,\text{Bq}$$
$$1\,\text{Curie} = 3.7 \times 10^{10}\,\text{decays/s} = 1\,\text{Ci}$$
$$1\,\mu\text{Curie} = 10 \times 10^{-6}\,\text{Ci} = 3.7 \times 10^{4}\,\text{decays/s} = 1\,\mu\text{Ci}$$

# 30.5: Half-Lives

Another approach: Given an initial number of nuclei $N_0$, how long do you have to wait until only half are left? Call that the half-life, $t_{1/2}$. An alternate way to write the exponential decay is

$$N(t) = N_0 \left(\frac{1}{2}\right)^{t/t_{1/2}}$$

This is the method the text uses in section 30.5 to introduce decay. The half-life and lifetime are related by

$$t_{1/2} = (\ln 2)\tau = 0.693\,\tau$$

It is important to recognize that the halflife does not depend on the original number, and that the activity $R(t)$ follows the same rule as the number $N(t)$. So, for example, the time it takes the activity to go from 100 Bq to 50 Bq is the same time it takes to go from 50 Bq to 25 Bq, or 40 Bq to 20 Bq, or any $R$ to $\frac{1}{2}R$.

## Carbon-14 Half-life

$^{14}$C is unstable, and decays via beta decay:

$$^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + \mathrm{e}^{-} + \bar{\nu}_e$$

The half-life is approximately $t_{1/2} = 5730\,\mathrm{yr}$, corresponding to a lifetime of $\tau = 8270\,\mathrm{yr}$. Suppose you intially had $m_0 = 1.00 \times 10^{-10}\,\mathrm{g}$ of $^{14}$C.

- How many nuclei is that?

$$N_0 = (1.00 \times 10^{-10}\,\mathrm{g}) \times \frac{6.022 \times 10^{23}\,\mathrm{atoms}}{1\,\mathrm{mole}} \times \frac{1\,\mathrm{mole}}{14\,\mathrm{g}}$$

$$N_0 = 4.301 \times 10^{12}\,\mathrm{nuclei}$$

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- What is the initial activity?

$$R_0 = \frac{N_0}{\tau} = \frac{4.30 \times 10^{12}}{8270\,\mathrm{yr}} \times \frac{1\,\mathrm{yr}}{3.1536 \times 10^{7}\,\mathrm{s}} = 16.5\,\mathrm{Bq}$$

(Note we had to convert from years to seconds to get Bq.)

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- What is the activity after 5730 yr?

$$R = R_0 \left(\frac{1}{2}\right)^{t/t_{1/2}}$$

$$R = 16.5\,\mathrm{Bq} \times \left(\frac{1}{2}\right)^{5730/5730} = 8.25\,\mathrm{Bq}$$

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- What is the activity after $t = 2t_{1/2} = 11\,460\,\mathrm{yr}$?

$$R = R_0 \left(\frac{1}{2}\right)^{t/t_{1/2}}$$

$$R = 16.5\,\mathrm{Bq} \times \left(\frac{1}{2}\right)^{11460/5730} = 4.12\,\mathrm{Bq}$$

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The half-life is approximately $t_{1/2} = 5730\,\mathrm{yr}$, corresponding to a lifetime of $\tau = 8270\,\mathrm{yr}$. Suppose you intially had $m_0 = 1.00 \times 10^{-10}\,\mathrm{g}$ of $^{14}$C.

- What is the activity after $t = 3t_{1/2} = 17\,190\,\mathrm{yr}$?

$$R = R_0 \left(\frac{1}{2}\right)^{t/t_{1/2}}$$

$$R = 16.5\,\mathrm{Bq} \times \left(\frac{1}{2}\right)^{17190/5730} = 2.06\,\mathrm{Bq}$$

## Carbon-14 Half-life

- How long do you have to wait until the expected activity is 1 decay/s? Here it is probably simpler to use the exponential form rather than the half-life form. We are looking for the time $t$ when $R = 1$.

$$
\begin{aligned}
R &= R_0 e^{-t/\tau} \\
1 &= R_0 e^{-t/\tau} \text{ Multiply both sides by } e^{t/\tau} \\
e^{t/\tau} &= R_0 \text{ Take natural logs of both sides} \\
\frac{t}{\tau} &= \ln R_0 \\
t &= \tau \ln R_0 = (8270\,\text{yr}) \times \ln(16.49) = \boxed{23\,180\,\text{yr}}
\end{aligned}
$$

# 30.6 Medical Applications of Nuclear Physics

The interaction of nuclear particles with matter is complicated. This section introduces a number of new terms and definitions. These are important in the field, but will not be on the final exam.

## What's Next?

- Examples and Applications
- Final Review