2.13 • **CALC** A turtle crawls along a straight line, which we'll call the x-axis with the positive direction to the right. The equation for the turtle's position as a function of time is $x\ (t)\ = 50.0\ \text{cm} + (2.00\ \text{cm/s})t - (0.0625\ \text{cm/s}^2)t^2$. (a) Find the turtle's initial velocity, initial position, and initial acceleration. (b) At what time t is the velocity of the turtle zero? (c) How long after starting does it take the turtle to return to its starting point? (d) At what times t is the turtle a distance of 10.0 cm from its starting point? What is the velocity (magnitude and direction) of the turtle at each of those times? (e) Sketch graphs of x versus t, v_x versus t, and a_x versus t, for the time interval $t = 0$ to $t = 40$ s.

The basic plan is to use the definitions:

$$v = \frac{dx}{dt}$$

$$a = \frac{dv}{dt}$$

where

$$x(t) = b + ct - et^2$$

where

$$b = 50.0\ \text{cm}$$
$$c = 2.0\ \text{cm/s}$$
$$e = 0.0625\ \text{cm/s}^2$$

(a) initial position

$$x(0) = b = \boxed{50.0\ \text{cm}}$$

$$v(t) = \frac{dx}{dt} = 0 + c - 2et$$

$$v(0) = 0 + \boxed{2.0\ \text{cm/s}}$$

2.13 (continued)

$$a(t) = \frac{dv}{dt} = -2e$$

$$a(0) = -2e = \boxed{-0.1250\ \text{cm}/s^2}$$

(b) when is $v = 0$?

$$v = c - 2et \quad , \text{set} = 0$$

$$0 = c - 2et$$

$$c = 2et$$

$$t = \frac{c}{2e} = \frac{2.0\ \text{cm}/s}{2(0.0625\ \text{cm}/s^2)}$$

$$\boxed{t = 16.0s}$$

(c) when does $x(t) = b$ again?

$$x(t) = b + ct - et^2$$

$$b = b + ct - et^2$$

$$et^2 = ct$$

$$t = \frac{c}{e} = \frac{2.0\ \text{cm}/s}{0.0625\ \text{cm}/s^2}$$

$$\boxed{t = 32.0s}$$

(The rest of this problem is more complicated than a test problem would be.)

2.13 (continued)

(d) when is $x(t) = 10.0$ cm from it's starting point?

i.e. when is $x(t) = b + 10.0 = 60.0$ cm
or $x(t) = b - 10.0 = 40.0$ cm?

Try $x = 60.0$ cm

$$60.0\,\text{cm} = 50.0\,\text{cm} + ct - et^2$$
$$0 = -10.0\,\text{cm} + ct - et^2$$
$$0 = -10.0\,\text{cm} + 2.0\,\text{cm/s}\ t - 0.0625\,\text{cm/s}^2\ t^2$$

Solve with the quadratic formula
(not on test)

$$t = \frac{\left(-2.0\,\text{cm/s} \pm \sqrt{\left(2.0\,\frac{\text{cm}}{\text{s}}\right)^2 - 4\cdot(10.0\,\text{cm})\left(0.0625\,\frac{\text{cm}}{\text{s}^2}\right)}\right)}{\left(2\cdot\left(-0.0625\,\frac{\text{cm}}{\text{s}^2}\right)\right)}$$

$t = 6.20$ s or 25.8 s

$$v(6.20\text{s}) = c - 2et$$
$$= 2.0\,\text{cm/s} - 2\left(0.0625\,\frac{\text{cm}}{\text{s}^2}\right)(6.20\text{s})$$

$v(6.20\text{s}) = 1.23$ cm/s
$v(25.8\text{s}) = -1.23$ cm/s

(2.13, continued)

If $x = 40.0\text{ cm}$,

$t = \underbrace{-4.40\text{ s}}$ or 36.4 s

ignore this — it is before the motion started.

$v(36.4\text{ s}) = -2.55\text{ cm/s}$

(e) back to

$$x(t) = 50.0\text{ cm} + 2.0\text{ cm/s}\ t - 0.0625\,\frac{\text{cm}}{\text{s}^2}\ t^2$$