Physics 151

Chapter 7 Notes—Part 1

Conservation of Energy

7 Potential Energy and Energy Conservation

We now have seen two different ways to approach a problem with constant forces and acceleration: $\sum \vec{F} = m\vec{a}$ and $K_i + W = K_f$. For constant force and acceleration, they are equivalent:

$$\begin{aligned} K_i + W &= K_f \\ \frac{1}{2}mv_i^2 + \sum F \cdot \Delta x &= \frac{1}{2}mv_f^2 \\ \frac{1}{2}mv_i^2 + (ma) \cdot \Delta x &= \frac{1}{2}mv_f^2 \\ \frac{1}{2}v_i^2 + a \cdot \Delta x &= \frac{1}{2}v_f^2 \\ v_i^2 + 2a \cdot \Delta x &= v_f^2 \end{aligned}$$

where the last line is familiar from our study of motion with constant acceleration.

But what if the forces (and hence acceleration) are *not* constant? While $\sum \vec{F} = m\vec{a}$ is still valid, it is less useful if we can't assume motion with constant acceleration.

Even if the acceleration is not constant, it is still true that

$$K_i + W = K_f$$

Further, for some forces, W can be determined just by knowing the initial and final positions, without having to calculate all the details in between. These forces are known as *conservative* forces, and are characterized by a *potential*, or stored energy.

7.1 Gravitational Potential Energy

Recall the example of lifting a a book of mass $m = 2.00\,\text{kg}$ up a height $h = 1.50\,\text{m}$ at constant speed. In doing so, you have to do $W = mgh = 29.4\,\text{J}$ of work, and gravity does $-29.4\,\text{J}$ of work. You can get that energy back. Suppose you drop the book and let it fall down 1.50 m. The book will accelerate in freefall. Just before it hits the table, it has velocity $v = \sqrt{2gh} = 5.42\,\mathrm{m/s}$ and kinetic energy $K = \frac{1}{2}mv^2 = 29.4\,\mathrm{J}$. During the fall, gravity does positive work $W = mgh = 29.4\,\mathrm{J}$ on the book.

We can put these ideas together to define a "Potential Energy" U for the gravitational force by

$$-(U_f - U_i) \equiv W$$

Rearranging the work-kinetic energy theorem gives another way to look at this.

$$\begin{aligned} K_i + W &= K_f \\ K_i - (U_f - U_i) &= K_f \\ K_i + U_i &= K_f + U_f \\ E_i &= E_f \end{aligned}$$

On the last line, we have written E for the total mechanical energy.

Note: Only the *change* in potential energy matters. We can choose the origin to be any convenient height—the floor, the table-top, the ceiling—as long as we are consistent and use the same origin for calculating the initial and final heights. The gravitational potential energy is thus:

$$U_g = mgy$$

This tells us how much energy is stored in a system by raising an object to a height y.

7.2 Elastic (Spring) Potential Energy

Stretching or compressing a spring takes work and can store energy. Recall Hooke's law for a spring:

$$F_s = -k(x - x_0)$$

where k is the spring constant, and x_0 is the relaxed position of the spring. We will usually try to set up our coordinate system so that $x_0 = 0$. Imagine stretching the spring from x_i to x_f (and assume for simplicity that $x_0 = 0$.) The work done is not simply $F \cdot \Delta x$ since the force F is not constant, but we can still calculate the work done by doing an integral. The change in potential energy is

$$\begin{aligned}
-(U_f - U_i) &= W \\
U_f - U_i &= -W = -\int_{x_i}^{x_f} F_s dx \\
U_f - U_i &= -\int_{x_i}^{x_f} (-kx) dx \\
U_f - U_i &= \int_{x_i}^{x_f} (kx) dx \\
U_f - U_i &= \frac{1}{2}kx^2\Big|_{x_i}^{x_f} \\
U_f - U_i &= \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2
\end{aligned}$$

Thus we will define **Elastic Potential Energy** by

$$U_s = \frac{1}{2}k(x - x_0)^2$$

Again, we will never have to do that integral—all we need to know are the initial and final positions of the spring.

7.3 Conservative and Non-conservative Forces

Caveat: The formulation in terms of potential energy only works if the work W depends only on the initial and final positions, and not on the path taken between them. For gravity and springs, this is true; for other forces, such as friction, it is not.

7.3.1 Conservative Forces

A force is called *conservative* if the work done depends only on the initial and final positions, and not on the path taken between them. Examples include gravity and springs. The work done in such a situation is *reversible*—worked stored as kinetic energy can be retrieved later on. To calculate work for conservative forces, you don't need to worry about the details of the path, you just need to know the initial and final positions.

7.3.2 Nonconservative Forces

A force is called *nonconservative* if the work done does depend on the path taken. Friction is a good example. If you slide a book across a table, the work done by friction is different if you take a straight path *vs.* a meandering path. To calculate work for nonconservative forces, you *do* need to worry about the details of the path.

Friction

The work done by friction depends on the path taken. There is no corresponding potential, or U value.

Thermal Energy What happens to the work done by friction? It increases the thermal energy of the system.

$$\Delta E_{th} = f_k \Delta x$$

Thus if friction is the only force acting on a system, you could write either of the following:

$$K_i + W_{\text{fr}} = K_f$$
$$K_i - f_k \Delta x = K_f$$

or

$$K_i = K_f + \Delta E_{\text{th}}$$

The first version will usually be more useful to us, until we get to the chapters on thermodynamics where we will be explicitly interested in tracking the thermal energy.

Other Forces

Other forces may also be involved, such as muscles or the crumpling of a fender during a car crash. We can't assume those are conservative unless we are explicitly given a U function for them.

7.4 Conservation of Energy

Starting with the general work-kinetic energy theorem:

$$K_i + W_{total} = K_f$$

we can break up the work into several different types. We will include work that is associated with a potential energy by including that potential. Work associated with friction can be included either as a loss in initial energy $-f_k\Delta x$ or as a increase in the final thermal energy ΔE_{th}. Other work (muscles, car fenders, *etc.*) we will simply include as W_{other}.

$$\begin{aligned}
K_i + W_{total} &= K_f \\
K_i + U_{gi} + U_{si} - f_k\Delta x + W_{other} &= K_f + U_{gf} + U_{sf} \\
K_i + U_{gi} + U_{si} - f_k\Delta x + W_{other} &= K_f + U_{gf} + U_{sf} \\
K_i + U_i - f_k\Delta x + W_{other} &= K_f + U_f
\end{aligned}$$

where on the last line we have grouped all the possible potential energies into a single U. This is a general form for the conservation of energy. In any particular problem, many of these terms may actually be zero.

Solving Energy Conservation Problems

Energy conservation problems typically involve comparing an initial state and a final state. The basic approach is rather methodical in nature:

1. Draw a good diagram labeling everything for both the initial and final states. With this diagram in hand, it will usually be easier to identify all the terms in the energy conservation equation and put the appropriate variables in.

2. Start with the general energy conservation equation:

$$K_i + U_i - f_k \Delta x + W_{other} = K_f + U_f$$

3. Consider the nonconservative terms $f_k \Delta x$ and W_{other}. Often, they will be zero. If not, you may have to do a Newton's Law problem to write those terms in terms of basic quantities given in the problem.
4. Write out in detail what each term means, while keeping the overal equation intact. That is, insert the formula for kinetic energy for each mass in the problem. Expand out the potential energy as appropriate to include gravity, springs, and any other potentials in the problem. Keep everything symbolic at first. Often, terms will cancel. Do this for both the initial state and final state.
5. Rearrange terms to isolate the unknown and solve.
6. Check your results for plausibility.

What's Next?

- Examples! Read the examples in the text, and see the examples posted on the course website.
- Force and Potential Energy (not on next test)
- Energy Diagrams (not on next test)