3. (20 pts.) A string that is under 50 N of tension has linear density 0.005 kg/m. A sinusoidal wave with amplitude 0.03 m and wavelength 2.0 m travels along the string. What is the maximum *transverse* speed of a particle on the string?

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It is important to distinguish between the speed of the wave and the up-and-down speed of the individual particles.

Wave speed: $v = \sqrt{F_T/\mu} = \sqrt{\dfrac{50\text{ N}}{0.005\text{ kg/m}}} = 100\text{ m/s}$

particle speed?

$$y = A\cos(kx - \omega t)$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{2.0\text{m}} = \pi/\text{m}.$$

$A = 0.03\text{m}$

$\omega = ?$ $\omega = 2\pi f$. Use $v = \lambda f$, so

$$f = \frac{v}{\lambda} = \frac{100\text{ m/s}}{2.0\text{ m}} = 50\text{ Hz}$$

Particles oscillate up and down in simple harmonic motion $y = A\cos(\omega t + \phi)$ at a fixed x.

The velocity is then $v_y = \underbrace{\dfrac{\partial y}{\partial t}}_{} = -\omega A\sin(\omega t + \phi)$

partial derivative means keep x fixed. Only vary t.

Max speed is when $\sin() = \pm 1$, so

$$v_{y,max} = \omega A = 2\pi f A$$
$$= 2\pi(50/\text{s})(0.03\text{ m}) = \boxed{9.42\text{ m/s}}$$

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$$y(x,t) = A \sin(kx - \omega t)$$

$$u_y = \frac{\partial y}{\partial t} = -\omega A \cos(kx - \omega t)$$

$$u_{y,max} = \omega A = 2\pi f A = 2\pi\left(\frac{v}{\lambda}\right)A$$

$$u_{y,max} = 2\pi\left(\frac{\sqrt{F_T/\mu}}{\lambda}\right)A = \boxed{9.42\ m/s}$$

This solution explicitly uses partial derivatives.