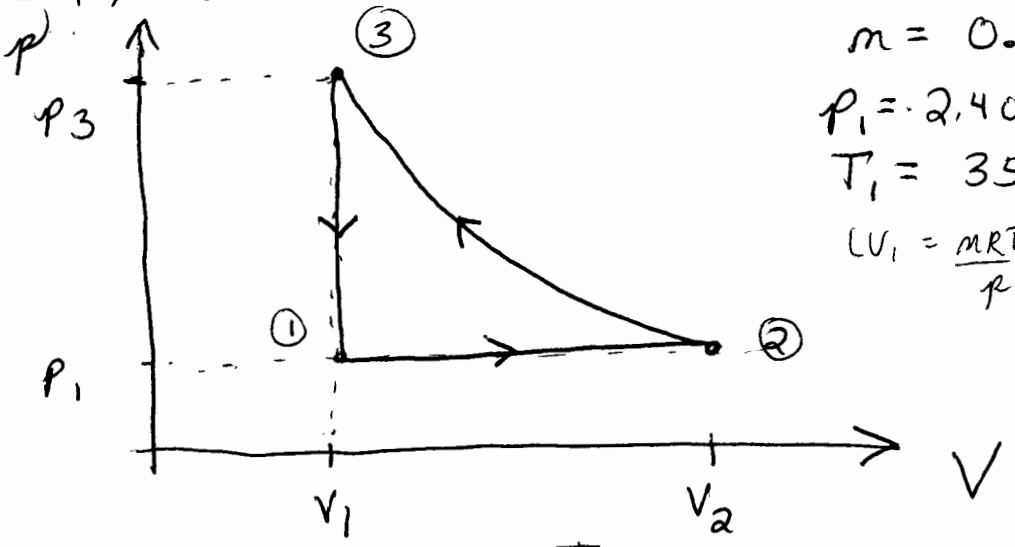
**17–46** A cylinder with a piston contains 0.250 mol of oxygen at $2.40 \times 10^5$ Pa and 355 K. The oxygen may be treated as an ideal gas. The gas first expands isobarically to twice its original volume. It is then compressed isothermally back to its original volume, and finally it is cooled isochorically to its original pressure. a) Show the series of processes on a $pV$-diagram. b) Compute the temperature during the isothermal compression. c) Compute the maximum pressure. d) Compute the total work done by the piston on the gas during the series of processes.

Example 17-46




$n = 0.250$ moles

$p_1 = 2.40 \times 10^5$ Pa

$T_1 = 355$ K

($V_1 = \frac{nRT_1}{p_1}$, if you insist)

Ideal gas: $pV = nRT$

oxygen - diatomic $C_V = \frac{5}{2}R$ $C_p = C_V + R = \frac{7}{2}R$.

(b) Fill out a table

| State | p | V | T | |
|---|---|---|---|---|
| 1 | $2.40 \times 10^5$ | $V_1$ | 355 | ↘ const p |
| 2 | $2.40 \times 10^5$ | $2V_1$ | 710 | ↘ const T |
| 3 | $4.80 \times 10^5$ | $V_1$ | 710 | ↘ const. V |
| 1 | $2.40 \times 10^5$ | $V_1$ | 355 | |

We need to fill in the boxes.

$T_2 = ?$ $\quad T_2 = \frac{p_2 V_2}{nR} = \frac{p_2}{nR}(2V_1) = 2\frac{p_2}{nR}\left(\frac{nRT_1}{p_1}\right)$

$T_2 = 2\frac{p_2}{p_1}T_1 = 2T_1 =$ **710 K**

↑ recall $p_2 = p_1$ !

(c) Find $p_3 = \frac{nRT_3}{V_3} = \frac{nRT_2}{V_1} = \frac{nRT_2}{nRT_1/p_1} = p_1\left(\frac{T_2}{T_1}\right)$

$p_3 = 2p_1 =$ **$4.80 \times 10^5$ Pa**

(d) Work

$$W_{12} = \int_1^2 p\,dV = p_1(V_2 - V_1) = p_1(2V_1 - V_1) = p_1 V_1 = nRT_1$$

$$\therefore W_{12} = nRT_1 = (0.250)(8.31)(355) = \underline{737.5\ \text{J}}$$

$$W_{23} = \int_2^3 p\,dV = \int_2^3 \frac{nRT}{V}\,dV = nRT_2 \ln\left(\frac{V_3}{V_2}\right)$$

$$W_{23} = (0.25)(8.31)(710)\ln\left(\frac{1}{2}\right) = \underline{-1022\ \text{J}}$$

$$W_{31} = 0 \quad (V = \text{constant})$$

$$W_{1231} = 737.5 - 1022 = \boxed{-284.5\ \text{J}}$$

Now find Q's

| Process | Q | W | $\Delta U$ |
|---|---|---|---|
| 1→2 | 2582 | 738 | 1844 |
| 2→3 | -1022 | -1022 | 0 |
| 3→1 | -1844 | 0 | -1844 |
| 1→2→3→1 | -284 | -284 | 0 |

1→2 (constant p). $Q_{12} = W_{12} + \Delta U_{12}$

$$\Delta U_{12} = n C_v (T_2 - T_1) = (0.250)\left(\frac{5}{2}R\right)(710 - 355)$$

$$\Delta U_{12} = 1844\ \text{J}$$

$$Q_{12} = W_{12} + \Delta U_{12} = 2581$$

2→3 $\Delta U_{23} = 0$ since $\Delta T = 0$.

$$\therefore Q_{23} = W_{23} + \Delta U_{23} = W_{23} = -1022$$

3→1 $Q_{31} = W_{31} + \Delta U_{31} = 0 + n C_v (T_1 - T_3)$

$$Q_{31} = -1844$$

NOTE: for 1231 cycle, $Q_{1231} = W_{1231} + \Delta U_{1231}$ with no round off.