Phys 335: Schroeder-02.23

Problem 2.23. Consider a two-state paramagnet with 10^{23} elementary dipoles, with the total energy fixed at zero so that exactly half the dipoles point up and half point down.

(a) How many microstates are "accessible" to this system?

(b) Suppose that the microstate of this system changes a billion times per second. How many microstates will it explore in ten billion years (the age of the universe)?

(c) Is it correct to say that, if you wait long enough, a system will eventually be found in every "accessible" microstate? Explain your answer, and discuss the meaning of the word "accessible."

2-state paramagnet, $N_\downarrow = N_\uparrow = \frac{1}{2}N$.

Start: $\Omega = \dfrac{N!}{N_\downarrow!\,(N-N_\downarrow)!} = \dfrac{N!}{\left((N/2)!\right)^2}$

Let $N \to$ large, e.g. 10^{23}

$$\Omega(N, N/2) \simeq \frac{N^N e^{-N}\sqrt{2\pi N}}{\left[(N/2)^{N/2} e^{-N/2}\sqrt{2\pi N/2}\right]^2}$$

$$= \frac{N^N e^{-N}\sqrt{2\pi N}}{(N/2)^N e^{-N}\ \pi N} = 2^N\sqrt{\frac{2}{\pi N}}$$

Note 2^N = total # of microstates including all $N_\downarrow$ values. But 2^N is a very large #, while $\sqrt{\frac{2}{\pi N}}$ is just 1/large and can be neglected.

OR.... The multiplicity function is very sharply peaked so that nearly all the states have $N_\uparrow = \frac{1}{2} N$.

How big is $2^{10^{23}}$?

$$\ln 2^{10^{23}} = 10^{23} \ln 2$$

(b) Explore 10^9 states / second for

$$t_{1yr} = 86400 \times 365.25 = 3.1 \times 10^7 \, s$$

$$n_{states} \simeq 10^9 \frac{states}{s} \times 3.1 \times 10^7 \frac{s}{yr} \times 10 \times 10^9 \, yr$$

$$= 3.1 \times 10^{25} \, s \approx 310 \times 10^{23}$$

Compare n_{states} to Ω

$$\frac{n_{states}}{\Omega} \sim \frac{3 \times 10^{25}}{2^{10^{23}}} \sim 0$$

no appreciable fraction of states are accessed.