Problem 2.35

At what temperature would the entropy of an ideal monatomic gas become negative? Consider helium originally at room temperature and atmospheric pressure. Cool it at constant density. At what temperature would the entropy go to zero? (Below that, it would become negative.)

In[18]:=
```
Clear["Global`*"]
```

Sackur-Tetrode Equation

In[19]:=
$$S[n\_, V\_, T\_] := N k \left(\text{Log}\left[\left(\frac{V}{n}\left(\frac{4\pi m U[n, T]}{3 n h^2}\right)^{3/2}\right)\right] + \frac{5}{2}\right)$$

In[20]:=
$$U[n\_, T] := \frac{3}{2} n k T \text{ (* For a monatomic ideal gas *)}$$

In[21]:=
```
S[n, V, T]
```

Out[21]=
$$k N \left(\frac{5}{2} + \text{Log}\left[\frac{2\sqrt{2}\,\pi^{3/2}\left(\frac{k m T}{h^2}\right)^{3/2} V}{n}\right]\right)$$

In[22]:=
```
Solve[S[n, V, T] == 0, T]
```

Solve : There may be values of the parameters for which some or all solutions are not valid.

Out[22]=
$$\left\{\left\{T \to \frac{h^2 n^{2/3}}{2 e^{5/3} k m \pi V^{2/3}}\right\}\right\}$$

In[23]:=
```
Tz = T1 /. %[[1]]
```

Out[23]=
$$\frac{h^2 n^{2/3}}{2 e^{5/3} k m \pi V^{2/3}}$$

In[24]:=
```
V = n k Ti / pi; (* Ideal gas law applied to initial conditions *)
```

In[25]:=
```
Tz
```

Out[25]=
$$\frac{h^2 n^{2/3}}{2 e^{5/3} k m \pi \left(\frac{k n Ti}{pi}\right)^{2/3}}$$

In[26]:=
```
Tz = FullSimplify[Tz, Assumptions → n > 0]
```

Out[26]=
$$\frac{h^2}{2 e^{5/3} k m \pi \left(\frac{k Ti}{pi}\right)^{2/3}}$$

In[27]:=
```
Ti = Quantity[300, "Kelvins"]
```
Out[27]=

300 K

In[28]:=
```
pi = Quantity[1, "Atmospheres"]
```
Out[28]=

1 atm

In[29]:=
```
k = Quantity[1, "BoltzmannConstant"]
```
Out[29]=

k

In[30]:=
```
h = Quantity[1, "PlanckConstant"]
```
Out[30]=

h

In[31]:=
```
m = ElementData["Helium", "AtomicMass"]
```
Out[31]=

4.002602 u

In[33]:=
```
Tz
```
Out[33]=

$0.0001675866\ \mathrm{atm}^{2/3}\mathrm{h}^2/(\mathrm{u}\ \mathrm{K}^{2/3}\mathrm{k}^{5/3})$

In[34]:=
```
UnitConvert[Tz, "Kelvins"]
```
Out[34]=

0.01212015 K