*The Two-Dimensional Ising Model*

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is the fact that he has implicitly assumed the most non-classical feature of quantum theory, namely that coordinates and momenta are incompatible observables. That is to say, he specifies a state by $q$ alone, or by $p$ alone, rather than by both $q$ and $p$.

Although the interpretation defended by these authors may be sound, Landé's attempt to derive the theory from intuitively acceptable principles has not succeeded.

#### References


1. See M. O. Scully, M. Sargent III, PHYSICS TODAY, March 1972, page 38. The August, December 1972 and February 1973 issues contain further correspondence on this subject.
2. A. Shimony, PHYSICS TODAY, September 1966, page 85. A review of *New Foundations of Quantum Mechanics* by A. Landé (Cambridge U. P., London, 1965).
3. A. Landé, *Foundation of Quantum Theory*, (Yale U. P., New Haven, 1955) pages 69–71.



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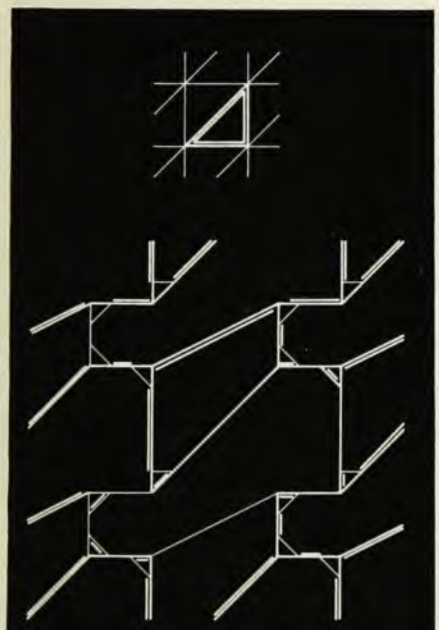
## The Two-Dimensional Ising Model

**B. M. McCoy, T. T. Wu**
418 pp. Harvard U.P., Cambridge, Mass., 1973. $25.00

One of the nicer ironies in the history of statistical mechanics is buried in the final section of E. Ising's analysis of W. Lenz's model of a ferromagnet. Having shown by explicit computation that the model has no ferromagnetic ordering in one dimension, Ising presents a simple "proof" that ordering cannot occur in higher dimensions either. Take as an explicit example the subject of the book under review here—the two-dimensional model with nearest-neighbor interactions on a square lattice: Suppose one were to increase the strength of the ferromagnetic coupling along the horizontal bonds, a process that can only increase any spontaneous magnetization. When the horizontal coupling becomes infinitely large, the spins in each horizontal row become locked together, and the problem reduces back to a one-dimensional array of two-state systems, where the spontaneous magnetization has already been proved to be zero at all temperatures.



**The one-to-one correspondence** between closed polygons on the counting lattice and closest-packed dimer configurations on the associated dimer lattice is shown. From the book, *The Two-Dimensional Ising Model.*

Apparently not everybody succumbed to this alluring argument, but a decade passed before R. E. Peierls outlined the proof that the two-dimensional model does indeed order. Only after another decade did L. Onsager publicly reveal the formula for the spontaneous magnetization.

This tale should be a cautionary one for those (and they probably include most physicists today) who feel in their hearts that care over the interchange of limits is best left to those whose concern for proper mathematical diction exceeds their interest in physical results. As everybody now knows, not only is there a transition in two dimensions, but the properties of the critical point revealed by Onsager have led to the revolution in our understanding of critical phenomena that still rages about us.

In spite of its vast impact, however, few physicists have savoured the delights of an intimate knowledge of the two-dimensional Ising model, because of the intricate and rather specialized mathematics that underlie exact computations. Now, however, two able practitioners of this arcane art have attempted to make its pleasures more widely available, by setting down, with great care, patience and good humor, a guide for the mathematically innocent to the exact results of Onsager and Yang, as well as to several more recent results (largely due to themselves) on boundary properties and the effects of random impurities. Because the authors' stated aim is to teach the reader all the necessary mathematics that is "not known by the average graduate student in physics," a non-trivial fraction of the volume is devoted to selected nuggets of pure mathematics (for example, Weiner–Hopf sum equations) presented in terms that the non-mathematician can actually hope to grasp. The approach to the subject is via Pfaffians, entities that the average graduate student in physics can barely pronounce,

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much less evaluate, which provides the occasion for more readable and accessible mathematics lessons.

As the authors freely acknowledge in their preface, even a book of more than 400 pages cannot deal completely with the subject, and such important topics as series expansions, existence and analyticity proofs, lattices other than square and applications other than magnetic phase transitions, are all omitted. I regret that a chapter was not added giving a short summary of such results, because without it the genuine neophyte will acquire a somewhat distorted view of the subject. The reader, for example, is told that efforts to find an exact solution in three dimensions have been so futile for so long as to render the endeavor not worth further serious effort; what is not mentioned, however, is that many important properties of the critical point in three dimensions have been computed numerically to impressively high accuracy.

Because it gives no broad overview of the subject, I suspect that the book will appeal to a rather limited audience. It will certainly now be required reading for any mathematicians or physicists wishing to join that intrepid band who are skilled enough to pry further secrets from the two-dimensional model. It will also, I would hope, be skimmed with pleasure by theoretical physicists (like me) who lack the skill to do such tricks, but enjoy catching a glimpse of how they are done.

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## Electronic and Ionic Impact Phenomena Vol. 4: Recombination and Fast Collisions of Heavy Particles

**H. S. W. Massey, H. B. Gilbody**
1008 pp. Oxford U. P., New York, 1974. $85.00

With this volume Sir Harrie Massey and Brian Gilbody complete the major part of the revision of an authoritative survey of electronic and ionic collision processes, whose first edition by Massey and Eric Burhop was published in 1952. (A fifth volume will include positron and muon collisions, together with recent advances since the closing dates of the various volumes, which for volume 4 was 1969–1970.) The importance of this book as a measure of the growth toward maturity of the large part of the whole field of atomic and molecular physics has been described in the review of volume 3 (PHYSICS TODAY, January 1973, page 77).

Massey is uniquely fitted to carry out the extraordinary task of reviewing the manifold accomplishments and present status of what has become a very large field of knowledge since he began working in it more than 40 years ago. A theoretician himself, he built up and still heads the physics department at University College London, which occupies a leading position in experimental as well as theoretical studies of collision phenomena. Gilbody, now at Queen's University, Belfast, is a product of Massey's department and a respected experimentalist in his own right.

This volume, and indeed the whole of the second edition, succeeds in carrying forward the aim of the first edition in describing the state of knowledge in this subject, emphasizing experimental techniques and results, and weaving them into an account based on a framework provided by theory. Especially valuable are the occasional summaries evaluating the various theoretical approximations as they compare with measured results.

Much of the material of this book consists of a compilation and recapitulation of the experimental results from a large fraction of the most important research papers in the field. Typically, the basic theory is presented and then the experimental technique is described in reasonable detail and illustrated by a diagram of the experimental arrangement used. Experimental results are presented with extensive use of figures from the original papers (600 figures are used in this volume of 1008 pages). While not pretending to be complete and exhaustive, the coverage of the literature up to 1969 is impressive, particularly for British and American publications.

The experimental descriptions and presentations of results are of such a quality that there will be a strong temptation to use this volume and its companions as a substitute for referring to the original literature. It is not intended as a criticism of this enormous and successful effort, but rather as a warning to the reader, that I point out that the authors, although exceedingly erudite and well organized, are not omniscient, and occasional (rare) omissions of significant papers can be detected. Certainly as a compilation of a great amount of material in one place, and as an entry into the literature of the field, these volumes will be uniquely useful to anyone working regularly in atomic-collision physics.

The principal material of this volume is devoted to fast collisions of heavy particles, preceded by a chapter on recombination. By recombination is meant the electron–ion and ion–ion processes of charge-neutralization and