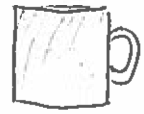
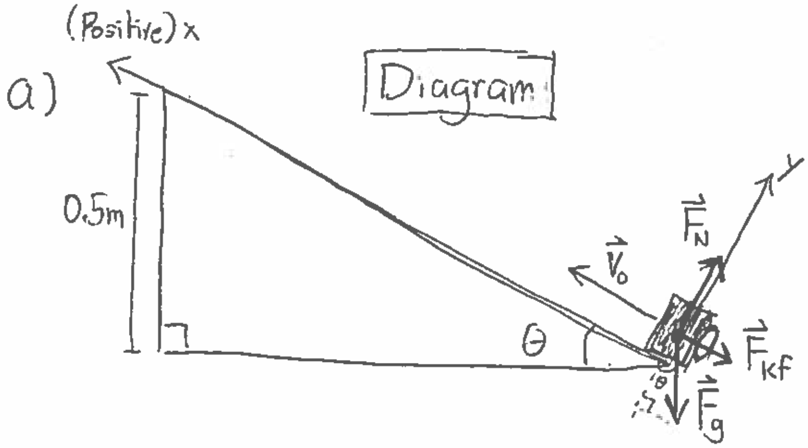
Sliding Up the Plane 

(Solution Set)

a)



We know that:

$$\begin{cases} \mu_k = 0.2 \\ \mu_s = 0.4 \\ V_o = 3.5\ m/s \\ \theta = 16^\circ \end{cases}$$

- To find how high the cup rises, we ed to know its acceleration: $\vec{F}_{net} = m\vec{a}$ from Newton's 2nd law, so we need to find $\vec{F}_{net}$

$\vec{F}_{net} = \sum_i \vec{F}_i = \vec{F}_N + \vec{F}_g + \vec{F}_{kf}$. Let's analyze the x and y components (see diagram above for the way the x and y axes are drawn) separately:

components: $\begin{cases} F_{net,x} = -mg\sin\theta - F_{kf}, \quad \text{where } F_{kf} = \mu_k F_N \\ F_{net,y} = F_N - mg\cos\theta = 0 \leftarrow \text{no motion in this direction} \end{cases}$

$$\longrightarrow F_N = mg\cos\theta.$$

- Plug this into $F_{kf} = \mu_k F_N = \mu_k mg\cos\theta$ in the other equation (for $F_{net,x}$)

$$F_{net,x} = -mg\sin\theta - \mu_k mg\cos\theta = ma_x$$

by Newton's 2nd law

$$\xrightarrow[\text{for } a_x]{\text{solve}} \underbrace{a_x = -g(\sin\theta + \mu_k\cos\theta)}$$

This is our (constant) acceleration in the x-direction.

- Now, plug this into the Kinematics eqns. for motion w/ constant acceleration:

kinematics eqns.
$$\begin{cases} x = \cancel{x_0}^{\,0} + v_{0,x}t + \frac{1}{2}a_x t^2 = v_0 t + \frac{1}{2}a_x t^2 \\ v = v_0 + a_x t \longrightarrow t = \frac{(v - v_0)}{a_x} \end{cases}$$

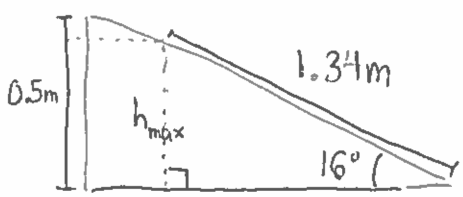
- At the highest point the cup reaches before stopping or turning around, $v = 0$, so $t_{top} = \frac{-v_0}{a_x}$ (which is the correct sign because $a_x < 0$). Plug this into eqn. for x:

$$x_{top} = v_0 t_{top} + \frac{1}{2}a_x t_{top}^2 = v_0\left(\frac{-v_0}{a_x}\right) + \frac{1}{2}a_x\left(\frac{-v_0}{a_x}\right)^2 = \frac{-v_0^2}{a_x} + \frac{1}{2}\frac{v_0^2}{a_x} = \frac{-v_0^2}{2a_x}$$

- so we have $x_{top} = \dfrac{v_0^2}{2g(\sin\theta + \mu_k\cos\theta)} = \dfrac{(3.5\,m/s)^2}{2(9.80\,m/s^2)[\sin(16°) + (0.2)\cos(16°)]}$

$$\approx 1.34\,m.$$

- This means that the height h_{max} to which the cup rises is



$\Rightarrow h_{max} = (1.34\,m)\sin(16°) \approx \boxed{0.368\,m}$

b) To determine whether the cup starts moving again, we need to know whether the applied force on the block overcomes static friction:

$$F_{app,x} = |F_{g,x}| = mg\sin\theta \quad \text{(because gravity's supplying this force)}$$

- The cup gets stuck when $F_{app,x} \le \mu_s F_N = \mu_s mg\cos\theta$

max angle before slipping occurs

$$\xrightarrow[\text{threshold}]{\text{at the}} F_{app,x} = mg\sin\theta_{max} = (mg\cos\theta)\mu_s \longrightarrow \mu_s = \frac{\sin\theta_{max}}{\cos\theta_{max}} = \tan\theta_{max}$$

so $\theta_{max} = \arctan(\mu_s) = \arctan(0.4) \approx 21.8°$

- Since $\theta = 16° < \theta_{max}$, the cup won't slip when static friction kicks in, so it **just "sticks" at h_{max} and doesn't slide down**.