Uniform Circular Motion (Solutions)

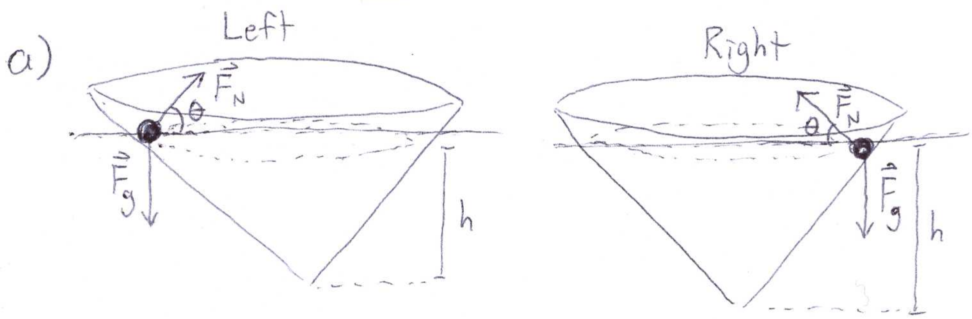
$\boxed{\text{Circles and Strings}}$: Tension provides the centripetal force here, so we have $F_T = Ma_{cent} = M\frac{V^2}{R}$, with the same v in each case; thus:

① $F_T^{①} = \frac{m}{r}v^2$ ③ $F_T^{③} = \frac{2m}{r}v^2$

② $F_T^{②} = \frac{m}{2r}v^2$ ④ $F_T^{④} = \frac{2m}{2r} = \frac{m}{r}v^2$

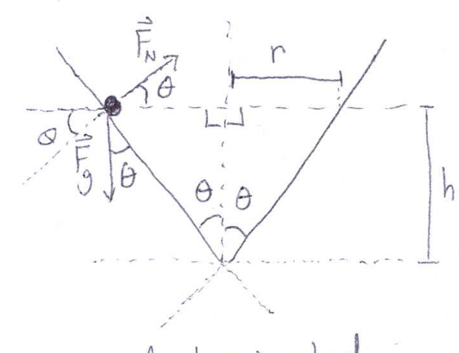
so, from largest to smallest: $\boxed{F_T^{③} > F_T^{①} = F_T^{④} > F_T^{②}}$

$\boxed{\text{Marble in a Cone}}$: a)



(b) This is uniform circular motion, so $F_{cent} = m\frac{v^2}{r}$

F_{cent} is the horizontal component of $\vec{F}_N$: $F_{cent} = F_N\cos\theta$



Angles involved

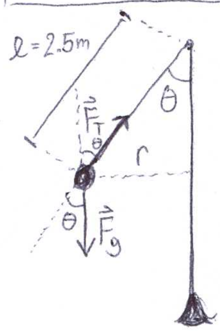
$F_N\sin\theta = mg \longrightarrow F_{cent} = \frac{mg\cos\theta}{\sin\theta} = \frac{mg}{\tan\theta}$

and $\tan\theta = \frac{r}{h} \longrightarrow r = h\tan\theta$, so

$\frac{mg}{\tan\theta} = \frac{mv^2}{h\tan\theta}$, or $\underbrace{v = \sqrt{gh}}$

$\boxed{\text{other than the height, no additional information is needed.}}$

Tetherball: $\ell = 2.5\text{m}$, $\theta = 40°$



a) The period is $\frac{2\pi r}{v} = T$, so we need r and v.

$r = \ell \sin\theta$

v can be found from $F_{cent} = m a_{cent} = m\frac{v^2}{r}$

$F_{cent} = F_T \sin\theta$, and $F_T \cos\theta = F_g = mg$ because the ball isn't moving in the vertical direction.

$$F_T = \frac{mg}{\cos\theta}, \quad a_{cent} = \frac{F_{cent}}{m} = \frac{F_T \sin\theta}{m} = \frac{g\cancel{m}\sin\theta}{\cancel{m}\cos\theta} = g\tan\theta$$

$$\text{so } g\tan\theta = \frac{v^2}{r} = \frac{v^2}{\ell\sin\theta} \longrightarrow v = \sqrt{g\ell\tan\theta\sin\theta}$$

$$\text{and } T = \frac{2\pi r}{v} = 2\pi\frac{\ell\sin\theta}{\sqrt{g\ell\tan\theta\sin\theta}} = 2\pi\sqrt{\frac{\ell}{g}}\sqrt{\frac{\sin\theta}{\tan\theta}}$$

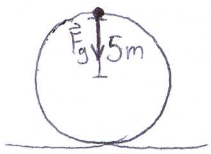
$$= 2\pi\sqrt{\frac{\ell\cos\theta}{g}} = 2\pi\sqrt{\frac{(2.5\text{m})\cos(40°)}{(9.8\,\text{m}/\text{s}^2)}} \approx \boxed{2.78\,\text{s}}$$

b) The impulse is equal to the change in momentum: $\vec{J} = \Delta\vec{p}$

$$|\vec{J}| = |\Delta p| = m\Delta v = m\sqrt{g\ell\tan\theta\sin\theta} \approx \boxed{1.818\,\frac{\text{m kg}}{\text{s}}}$$

And the direction is $\boxed{\text{tangential}}$ to the circle the tetherball traces out as it moves.

Rollercoaster Engineering:



The minimum velocity the cars must have at the top of the arc to remain in circular motion is given by $F_{cent} = \frac{v^2}{r} m = mg$, i.e., when the centripetal force is provided by gravity alone: $v = \sqrt{gr}$. The associated kinetic energy $K = \frac{1}{2} m v^2$ must come from the initial potential energy $U = mgh \longrightarrow K = \frac{1}{2} m v^2 = \frac{1}{2} mgr = mgh \longrightarrow h = \frac{r}{2}$

or $\boxed{h = 2.5\,\mathrm{m}}$